Economics 202 — Matthias Doepke
Macroeconomics — Winter 2008

Problem Set 4

Voting on Public Good Provision

Consider an economy populated by three different groups of equal size, the poor (P), the middle class (M), and the rich (R). The three groups are distinguished by income, where we have:

$$y_P < y_M < y_R,$$

Preferences are defined over consumption c and a public good g:

$$u(c, g) = \frac{c^{1-\sigma}}{1-\sigma} + \frac{g^{1-\sigma}}{1-\sigma}.$$

The government finances the public good g by levying a proportional income tax τ. An individual's budget constraint is given by:

$$c = (1 - \tau)y_i,$$

and the government's budget constraint is:

$$g = \tau\bar{y},$$

where $\bar{y}$ is average income.

Question 1:
(a) Show that preferences over taxation (or public good provision) are single peaked in this economy.

(b) Define a majority-voting equilibrium.

(c) Find the tax rate and the level of g that would prevail under majority voting in this economy.

(d) Now assume that voters have preferences over political parties that are independent of the chosen policy. Also, assume that the distribution of party bias is identical in the three groups. Define and compute a probabilistic voting equilibrium, and provide a condition on the y_i under which probabilistic voting leads to the same outcome as majority voting.

Dynamic Voting

Consider the following dynamic extension of the economy considered in the previous question. This time, there are only two groups in the economy, the poor and the rich, with fixed income levels $y_P < y_R$. The poor make up fraction p of the population, and the rich are fraction $1 - p$. of the As before, preferences are defined over consumption c and a public good g, where the government finances the public good g by levying a proportional income tax τ. The budget constraints are unchanged. The new feature of the economy is that the probability of being rich in the next period (which is assumed to be identical for rich and poor people for simplicity) also depends on the tax rate. One interpretation of this relationship is that the probability of being rich depends on investment in education, and high taxes discourage investing in education. However, rather than modeling the education choice in detail, for simplicity we impose a particular functional form for this relationship. The probability of being poor in the next period is given by (and hence the fraction of poor people p' in the next period):

$$p' = \frac{1}{2} + \frac{1}{2}\tau^2.$$

Thus, the poor will always be in the majority. Agents live forever and discount future utility with discount factor β. The only state variable for the economy is the fraction of poor agents p, which is restricted to lie in the interval $[0.5, 1]$. Given a policy rule $\tau = \Psi(p)$ and assuming log utility, the value functions of poor and rich agents $i \in \{P, R\}$ can be written as:

$$v_i(p, \Psi) = \frac{((1 - \Psi(p))y_P)^{1-\sigma}}{1 - \sigma} + \frac{(\Psi(p)\bar{y})^{1-\sigma}}{1 - \sigma} + \beta(p' v_P(p', \Psi) + (1 - p')v_R(p', \Psi)),$$

where:

$$p' = \frac{1}{2} + \frac{1}{2}\Psi(p)^2$$

and $\bar{y} = p y_P + (1 - p)y_H$. Notice that the Bellman equation does not have a max operator because there is no individual decision variable. Voters also have to be able to evaluate the consequences of tax choices that are not prescribed by the policy rule Ψ. The utility of a poor voter if today's tax is τ and the future policy rule is Ψ is given by:

$$\tilde{v}_P(p, \tau, \Psi) = \frac{((1 - \tau)y_P)^{1-\sigma}}{1 - \sigma} + \frac{(\tau\bar{y})^{1-\sigma}}{1 - \sigma} + \beta(p' v_P(p', \Psi) + (1 - p')v_R(p', \Psi)),$$

where:

$$p' = \frac{1}{2} + \frac{1}{2}\tau^2.$$

Policies are determined by majority voting, which implies that the poor voters decide:

$$\tau = \underset{\tau}{\operatorname{argmax}}\{\tilde{v}_P(p, \tau, \Psi)\}.$$

Question 2:
(a) Define a politico-economic equilibrium for this economy.

(b) Compute a politico-economic equilibrium for an economy with parameters $y_P = 1$, $y_R = 10$, $\beta = 0.8$, $\sigma = 2$. Use a discrete grid for the the political choice variable τ (in the interval 0.01 to 0.99) and a corresponding discrete grid for the state variable p. Use a constant tax τ as your initial guess Ψ_0 for the policy rule. Then, solve for the value functions by value-function iteration, and generate a new guess for Ψ_1 by setting $\Psi_1(p) = \text{argmax}_\tau\{\tilde{v}_P(p, \tau, \Psi_0)\}$. Continue this process until convergence. If the algorithm does not convergence, stop after 20 iterations. Plot the initial guess Ψ_0, the first iteration Ψ_1, and the final iteration Ψ_{20} (or lower than 20 if your algorithm converges faster).

(c) Compare the optimal policy rule in the static and the dynamic settings, and explain how and why they are different.

The AK Model

Consider an economy in which the representative consumer has preferences:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma},$$

and in which the aggregate resource constraint is given by:

$$c_t + k_{t+1} = (1-\delta)k_t + Ak_t.$$

Question 3:
(a) Define a competitive equilibrium for this economy.

(b) Formulate the Bellman equation for the social planning problem, and solve the Bellman equation using guess-and-verify, with the guess:

$$v(k) = \alpha \frac{k^{1-\sigma}}{1-\sigma}.$$

(c) Compute the growth rate of the economy.

Growth with Human and Physical Capital

Consider an economy in which the representative consumer has preferences:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}.$$

The production function is given by:

$$y_t = A k_t^{\alpha} h_t^{1-\alpha}.$$

The laws of motion for physical and human capital are:

$$\begin{aligned} k_{t+1} &= (1-\delta)k_t + x_{kt}, \\ h_{t+1} &= (1-\delta)h_t + x_{ht}. \end{aligned}$$

Output can be converted one-to-one into consumption, physical investment, or human-capital investment.

Question 4:
(a) Define a competitive equilibrium for this economy, and formulate the social planning problem.

(b) Formulate the Bellman equation for the social planning problem, and show that the value function is homogeneous (of which degree?).

(c) Compute the balanced growth rate of the economy, and characterize the transitional dynamics for arbitrary initial levels of k_0 and h_0.