Econ 201A Homework 4

This homework is to be handed in on the last Tuesday class. This will give you a chance to raise questions in the TA section.

1. Certainty Equivalent

An individual with certain wealth *w* can either pay α to avoid facing a risk or accept a risky alternative $\tilde{x}$ so that his consumption is $\tilde{c} = w + \tilde{x}$ where *x* has mean 0 and variance σ^2. If the risk is small so that the second order Taylor's Expansion is a good approximation, obtain an expression for the "certainty equivalent" α and show that it is a function of the individual's degree of absolute risk aversion. Note: You must expand around $c = w$ so that the higher order terms are indeed small for small *x*. (Thank you Roberto.)

2. Equilibrium with CES preferences and identical beliefs.

In an economy with one commodity and S states all individuals have the same CES expected utility function and identical beliefs, that is $U(c) = \sum_{s=1}^{S} \pi_s v(c_s)$ where $v'(c_s) = c_s^{-R}$.

(a) Prove that the WE state claims price vector *p* satisfies the following condition $\frac{p_s}{p_t} > \frac{\pi_s}{\pi_t}$ where the total endowment of state s claims $\omega_s < \omega_t$.

(b) Explain why each individual, regardless of his initial holdings of assets, will consume a fraction of the aggregate endowment.

(c) Does it therefore follow that even if there are more states than assets, individuals can achieve the same outcome by trading only in the asset markets?

(d) Suppose that there is production in the model. Does your argument extend to this case as well?

3. State Claims and Asset Market Equilibrium

Consider a two state economy with one commodity. There are 10 plantations that face no risk. Each produces 10 units in each state. There are 20 identical plantations affected by the rainfall on the rainy side of the island. Each produces 15 units in state 1 and 5 in state 2. The probability of state 1 is one third. There are 30 agents in this economy and each initially owns one plantation. Each has VNM utility function $v(c) = -c^{-1}$.

(a) Solve for the Walrasian Equilibrium state claims prices.

(b) Hence solve for the implied asset prices.

(c) Consider a representative agent trading only in asset markets. Let q_A the number of riskless plantations that she purchases and let q_B be the number of risky plantations purchased. Since she is the only agent her initial asset holding is $(\overline{q}_A, \overline{q}_B) = (10, 20)$. Show that her expected utility is

$$U(q) = \pi_1 v(10q_A + 15q_B) + \pi_2 v(10q_A + 5q_B)$$

(d) Write down her portfolio constraint and show that the FOC is

$$MRS(q) = \frac{\pi_1 10 + \pi_2 10 \dfrac{v'(10q_A + 5q_B)}{v'(10q_A + 15q_B)}}{\pi_1 15 + \pi_2 5 \dfrac{v'(10q_A + 5q_B)}{v'(10q_A + 15q_B)}} = \frac{P_A^{asset}}{P_B^{asset}}.$$

(e) Appeal to market clearing and hence solve for the equilibrium prices in the asset market.
(f) Explain carefully the relationship between your answers to (b) and (e).

4. State Claims Market Equilibrium

Alex and Bev have the same expected utility function

$$U(c^h, \pi) = \pi_1 \ln(2 + c_1^h) + \pi_2 \ln(2 + c_2^h)$$

They live on an island where the rainfall can be high (good) or low (bad). There are two coconut plantations on the island. On plantation A the return is (4,9) and on plantation B the return is (12, 27) so that the total supply of coconuts is 16 in the bad state (s=1) and 36 in the good state (s=2). Alex owns the first plantation and Bev the second. The two states are equally likely.

(a) Solve for the PE allocations and hence show that $p = (2,1)$ is an equilibrium state claims price vector.
(b) Solve for Alex's equilibrium consumption in each state.
(c) Explain why trading shares in the two plantations does not help in the reallocation of risk.

Suppose plantation A issues two classes of stock. The total issue of preferred stock pays a dividend of 3 in each state. The ordinary stock pays the remaining return as dividend. Thus anyone can buy a share in the total return of the firm by buying the same quantity of the two classes of stock.

(d) With contingent markets, what will be the total market value of each class of stock?
(e) Comment on whether trading in assets is now a perfect substitute for trading in state claims markets.

5. Time and Uncertainty

The total endowment of two commodities in period 1 is $(\omega_{11}, \omega_{12}) = (81, 36)$. There are two states in period 2. If s =1 the total endowment of the two commodities is $(\omega_{(1)21}, \omega_{(1)22}) = (100, 25)$. If s=2 the total endowment is $(\omega_{(2)21}, \omega_{(2)22}) = (49, 121)$. Goods are not storable.

The representative agent has lifetime expected utility $U(c) = v(c_1) + \sum_{s=1}^{2} \pi_s v(c_{(s)2})$ where

$$v(c_1, c_2) = c_1^{1/2} + 2c_2^{1/2}.$$

State 1 is twice as likely as state 2.

(a) Solve for the WE state claims price vector. Note: $p_{(s)2j}$ is the price of a claim to a unit of commodity *j* to be delivered in period 2 if the state is s.
(b) A new technology is invented and placed in the public domain. Using the technology, each unit of commodity 2 in period 1 can be transformed into 1 unit of commodity 2 in period 2 in both states. What is the profit of the firm if it transforms q units? (Note: The technology is costless storage.)
(c) What is the new Walrasian Equilibrium?
(d) If a technology firm is invented that will costlessly store commodity 1 would this be profitable? Explain carefully.
(e) If there is storage what can you say about the relationship between the equilibrium price of commodity 1 in period 1 and the state claims prices of this commodity for delivery in period 2?
(f) If the spot market were to open in state s, what would be the relative price of commodity 1?
(g) Would trading in the spot market, assets and future spot markets be a perfect substitute for trading in contingent markets? You should assume that there are two "firms" one producing all of commodity 1 in each period and state and the other all of commodity 2. (A brief comment is all that is expected!)