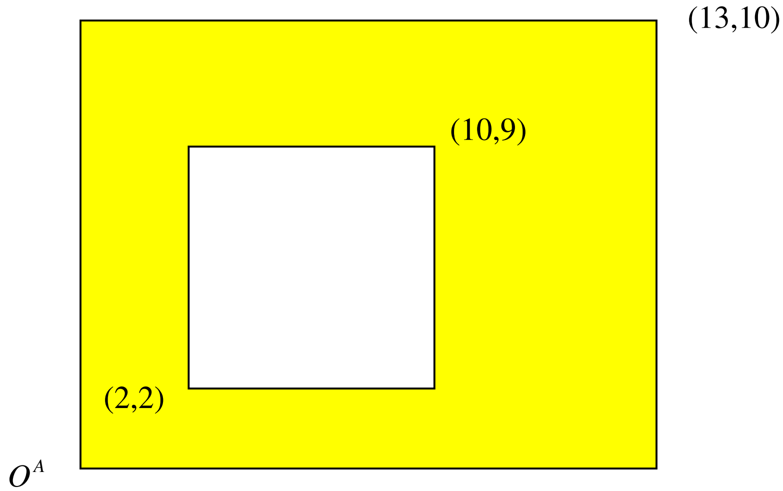
Econ 201 A Homework 3 Due Nov 14 (corrected)

1. Walrasian Equilibrium Prices

Alex has an endowment vector ω^A and utility function $U^A(x^A) = (x_1^A - \alpha_1^A)^2(x_2^A - \alpha_2^A)$. Bev has an endowment vector ω^B and utility function $U^B(x^B) = (x_1^B - \alpha_1^B)(x_2^B - \alpha_2^B)^2$. Note that Alex has a minimum consumption of α^A while Bev has a minimum consumption of α^B. The total endowment is $\omega = (13,10)$. Thus if $\alpha^A = (2,2)$ and $\alpha^B = (3,1)$, in order to survive, the allocation must be in the unshaded region of the Edgeworth Box depicted below.



(a) Explain why the PE allocations must lie below the diagonal joining the corners of the unshaded region.

HINT: You might define the "net consumption vector $c^h = x^h - \alpha^h$ and net endowment vector $\bar{\omega}^h = \omega^h - \alpha^h$, $h = A, B$ and so convert the model to a standard Cobb-Douglas model inside the survival region..

(b) What is the range of possible WE price ratios in this economy?

(c) If $\omega^A = (8,5)$ and $\omega^B = (5,5)$ solve for the WE price ratio.

2. Walasian Equilibria

Suppose that the minimum consumption vectors are instead $\alpha^A = (4,6)$ and $\alpha^B = (7,3)$.

(a) Explain why both will survive as long as the price ratio p_1 / p_2 is between ¼ and 1.

Henceforth set the price of commodity 2 equal to 1.

(b) Obtain an expression for the excess market demand for commodity 1,

$e_1(p_1) = x_1(p_1) - \omega_1 = c_1 - \bar{\omega}_1$.

(a) Hence show that $p_1 = 1/4$ and $p_1 = 1$ are both equilibrium prices. Depict the two WE allocations and budget lines in an Edgeworth Box diagram.
(b) Are there other WE prices of commodity 1 as well?
(c) Suppose that there is a reallocation of the endowments so that $\omega^A = (10,4)$ and $\omega^B = (3,6)$. Show that the equilibrium price of commodity 1 is ½.
(c) Depict the market demand curve and the supply curves in a neat figure with p_1 on the vertical axis. Anything puzzling?

3. Production Efficiency

There are two firms, $f = 1,2$. Firm *f* has a production set Y^f that is convex and satisfies the free disposal property.
(a) Show that the aggregate production set $Y = Y^1 + Y^2$ is convex.
(b) Does it also satisfy the free disposal property?
A production vector $\hat{y} \in Y$ is efficient if there is no other $y' \in Y$ such that $y' \geq \hat{y}$. Thus, by the supporting hyperplane theorem there exists a vector *p* such that, for any efficient production vector $\hat{y}$,

$$p \cdot y \leq p \cdot \hat{y} \text{ for all } y \in Y$$

(c) Since $\hat{y}$ is in the production set there must be some $\hat{y}^1 \in Y^1$ and $\hat{y}^2 \in Y^2$ such that $\hat{y} = \hat{y}^1 + \hat{y}^2$. Prove that $p \cdot y^f \leq p \cdot \hat{y}^f$, $f = 1,2$.

4. Aggregate Production sets

Firm 1 has a production set

$$Y^f = \{(z^f, q^f) \mid q^f \leq \alpha^f z^f - \beta^f (z^f)^2, \ f = 1,...,F. \ \alpha^f, \beta^f \geq 0.$$

(a) If there are 2 identical firms so that $\alpha^f = \alpha$ and $\beta^f = \beta$, what is the aggregate production set?
(b) If there are *n* identical firms what is the aggregate production set. Depict it in a neat figure for different values of *n* and describe what happens as *n* gets large.
(c) If the input and output prices are *r* and *p*, solve for the industry supply curve, $q = f(p/r)$
(d) Suppose that $\alpha^1 > \alpha^2 > 0$ and $\beta^1 > 0 = \beta^2$. Completely characterize the aggregate production set.
(e) Without any mathematics, explain what the aggregate production set must look like if the industry includes firm 1 and many identical firms each with a production set

$$Y^f = \{(z^f, q^f) \mid q^f \leq \alpha z^f - \beta (z^f)^2, \text{ where } \alpha^1 > \alpha > 0$$