

## Homework #2

### 1. Production and Cost

If a firm has production function  $q = (z_1 + 2z_2^{1/2})^2$  and the input price vector  $r = (4, 2)$  solve for the firm's total cost, average cost and marginal cost functions. Are they all differentiable for all positive output levels?

### 2. CES and CES like cost functions

A firm has production function  $q = ((a_1 z_1)^{\frac{\sigma-1}{\sigma}} + (a_2 z_2)^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ ,  $0 < \sigma$ ,  $\sigma \neq 1$ . The input price vector is  $r$ .

- (a) Confirm that the production function is strictly quasi-concave.
- (b) Show that the firm's cost function can be obtained by solving the following problem.

$$\text{Min}_y \left\{ \sum_{j=1}^2 \left( \frac{r_j}{a_j} \right) y_j \mid \sum_{j=1}^2 y_j^{\frac{\sigma-1}{\sigma}} = \text{const} \right\} \text{ where } q = (y_1^{\frac{\sigma-1}{\sigma}} + y_2^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}.$$

Hence or otherwise solve for the cost function.

- (c) A firm has production function  $q = \frac{z_3}{\left(\frac{1}{z_1} + \frac{1}{z_2}\right)}$ . By setting  $\sigma = 1/2$  in part (b) or otherwise,

solve for the firm's cost function.

### 3. Equilibrium and Time in an economy with no production

All consumers have the same utility function  $U = \sum_{t=1}^T \delta^{t-1} u(x_t^h)$ ,  $h = 1, \dots, H$  where  $x_t^h = (x_{t1}^h, x_{t2}^h)$

and  $u(x_t^h) = \ln x_{t1}^h + 2 \ln x_{t2}^h$ . The discount factor is  $\delta = 0.8$ . Initially  $T = 2$  and the aggregate endowment is  $\omega = (10, 20, 6, 12)$ .

- (a) Show that preferences are homothetic.
- (a) Solve for the Walrasian Equilibrium (WE) price vector if consumers trade in spot and futures markets.
- (b) Solve for the market interest rate and future spot price of commodity 2 if the spot and future spot market price of commodity 1 is 1.

- (c) Repeat your analysis if the endowment vector is instead  $\omega = (10, 20, 6, 6)$ .
- (d) In the light of your answers, comment on the following.

“A WE with interest rates and future spot markets but no futures markets requires all consumers to know how the proportional drop in the endowment of commodity 2 will affect prices in the future.”

- (e) Henceforth return to the endowment of part (b). Suppose that commodity 1 (but not commodity 2) can be stored without any deterioration. How much of the commodity will the representative agent wish to store?
- (f) What are the new WE spot and futures prices?
- (g) What is the equilibrium market interest rate if the spot and future spot price of commodity 1 is 1?
- (h) Repeat part (b) if there are three periods and the aggregate endowment is  $\omega = (10, 20, 6, 12, 5, 10)$ .
- (i) Also repeat part (c).