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Econ 201A

Homework 1 Fall 2006

1. Modified Cobb-Douglas Consumers

Alex has the following utility function $U(x)=(b_1+x_1)^{\alpha_1}(b_2+x_2)^{\alpha_2}(b_3+x_3)^{\alpha_3}$. The price vector is p and income is I.

(a) Solve for Alex's demand function assuming that consumption of all three commodities is strictly positive.

(b) Show that a necessary and sufficient condition for strictly positive consumption can be written in the following form.

$$b_j p_j \le \theta_j (b \cdot p + I), \quad j = 1,2,3.$$

You must determine the parameter vector $\theta = (\theta_1, \theta_2, \theta_3)$

Henceforth assume that $\alpha = p = (1,1,1)$.

(c) If $b = (3,4,5)$, for what income levels, if any, will Alex consume only one commodity?

(c) If $b = (b_1, 4, 5)$, for what values of b_1 will Alex consume only 1 commodity as long as income is sufficiently small.

2. Which Commodities?

Bev has the following utility function $U(x) = x_1^{\alpha} x_2^{\beta} + x_1^{\alpha} x_3^{\beta}$. The price vector is p and income is I.

(a) If $\beta = 1$ solve for the demand functions for all possible price vectors.

(b) If $\beta = 2$ again solve for the demand functions for all possible price vectors.

3. One step at a time

Ned has the following utility function $U = x_1^{1/2} + x_2^{1/2}$. The price vector is p and income is I.

(a) Solve for his maximized utility and show that it can be written in the form $U(x^*) = A(p) I^{1/2}$.

(b) Suppose instead that his utility function is $U(x) = (x_1^{1/2} + x_2^{1/2})x_3^{1/2}$. Why must it be the case that Ned will consume a strictly positive amount of each commodity? Solve for his demand functions.

HINT: One approach is to first allocate y dollars to be spent on commodities 1 and 2 and then appeal to your answer to (a) to solve for the optimal consumption $(x_2(y), x_3(y))$. Then substitute back into the utility function to get a derived utility function $u(y, x_3) = U(x_1(y), x_2(y), x_3)$. Finally note that $y + p_3 x_3 = I$

4. Exponential Consumer

Tom has a utility function $U(x) = -b_1 e^{-Ax_1} - b_2 e^{-Ax_2}$. The price vector is p and income is *I*.

(a) Solve for the demand functions assuming that consumption of both commodities is strictly positive.

(b) Under what conditions, if any, will consumption of commodity 1 be zero?

(c) Under what conditions, if any, will consumption of commodity 2 be zero?

5. Pareto Efficiency

In an economy there are 100 units of commodity 1 and 100 units of commodity 2. As the benevolent dictator you wish to choose some utility level for Bev and then choose the allocation for Alex that maximizes her utility. Such an allocation is said to be Pareto Efficient. This can be illustrated in an Edgeworth Box diagram (see below) where the bottom left and top right corners are the zero consumption points for each consumer.

(a) If both Alex and Bev have the same Cobb-Douglas preferences $U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$, where α and β are strictly positive, solve for the Pareto Efficient allocations. Depict these allocations in an Edgeworth box diagram.

Henceforth assume that Alex has a utility function $U^A = 2\ln x_1^A + x_2^A$ and Bev has a utility function $U^B = 6\ln x_1^B + x_2^B$.

(b) Solve for the Pareto Efficient allocations in which both consumers consume a strictly positive amount of each commodity.

(c) Show that this will be the case if $\ln 75 < \bar{U}^B < \ln 75 + 100$.

(d) What are the Pareto Efficient allocations if these inequalities are not satisfied? Derive your result. Depict all the PE allocations in an Edgeworth box.