Economics 201A, Midterm Review — Anton Cheremukhin

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1. WE and PO in the Edgeworth box

Exercise 1 (MT99, Q1) *Alex has utility* $U^A = -\frac{1}{x_1} - \frac{1}{x_2}$ *and endowment* $w^A = (6, 18)$

Bev has utility $U^B = -\frac{1}{y_1} - \frac{1}{y_2+10}$ *and endowment* $w^B = (4, 12)$

a) Characterize as completely as you can the PO allocations.

b) What is the WE price vector?

c) What are the possible WE price vectors, if the initial endowments change, leaving the aggregate endowment fixed?

a) $U^A = -\frac{1}{x_1} - \frac{1}{x_2} \to \max_x \quad s.t. \quad U^B = -\frac{1}{y_1} - \frac{1}{y_2+10} \geq U$

$s.t. \quad x_1 + y_1 \leq w_1 = 4 + 6 = 10 \quad s.t. \quad x_2 + y_2 \leq w_2 = 18 + 12 = 30$

$L = -\frac{1}{x_1} - \frac{1}{x_2} + \lambda \left(-\frac{1}{10-x_1} - \frac{1}{30-x_2+10} - U \right)$

$\text{FOC}_{x_1} : \frac{1}{(x_1)^2} = \lambda \frac{1}{(10-x_1)^2} \qquad \text{FOC}_{x_2} : \frac{1}{(x_2)^2} = \lambda \frac{1}{(40-x_2)^2}$

PO points in the interior: $\frac{x_1}{10-x_1} = \frac{x_2}{40-x_2} \quad \Leftrightarrow \quad \boxed{x_2 = 4x_1}$

This line is in the interior for $0 < x_1 < 7.5$ and $0 < x_2 < 30$.

For $10 \geq x_1 \geq 7.5$ it must be $x_2 = 30$ because that's the RC of the economy.

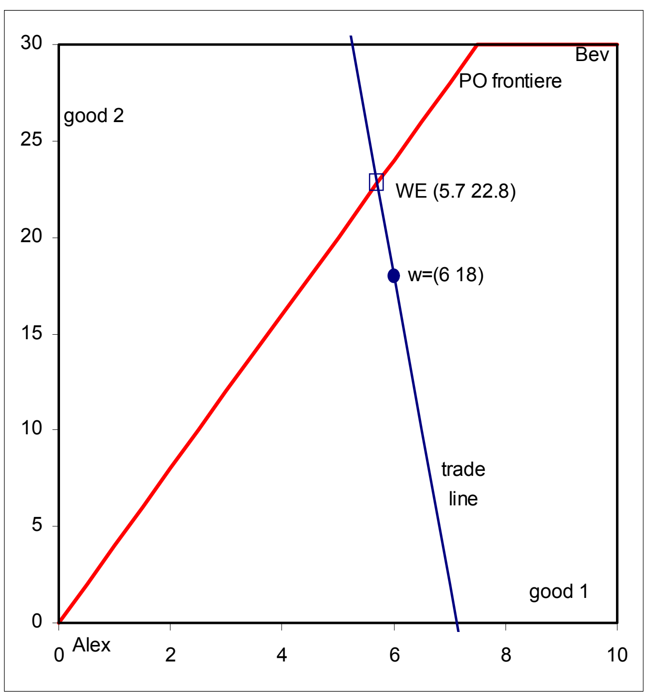
b-c) $MU_1^A = (x_1)^{-2} \qquad MU_2^A = (x_2)^{-2} \qquad p_1/p_2 = MU_1^A / MU_2^A = \left(\frac{x_2}{x_1} \right)^2$

For the interior case: $\quad p_1/p_2 = 16$

For the corner case: $\quad p_1/p_2 = \left(\frac{30}{x_1} \right)^2 |_{x_1 \in [7.5, 10]} \in [9, 16]$

The WE is $16x_1 + x_2 = 16 * 6 + 18 \quad s.t. \quad x_2 = 4x_1$

$20x_1 = 114 \qquad x_1 = \frac{57}{10} \qquad x_2 = 4 * \frac{57}{10} = \frac{114}{5} = 22.8$



2. **Robinson Crusoe**

Exercise 2 (MT04, Q3) *Robinson the manager operates a firm with production set*
$Y = \{(L, C)\,|1000L^2 - C^3 \geq 0, L \geq 0\}$
Crusoe the consumer has a utility function $U(L, C) = 3\ln C + 2\ln(54 - L)$
a) Solve for the firm's supply function
b) Solve for Robinsons' demand.
c) Find the WE wage/price ratio

a) $\pi(p, w) = \max_{(C,L)} \{pC - wL | 1000L^2 - C^3 \geq 0, L \geq 0\}$
$\pounds = pC - wL + \lambda(1000L^2 - C^3)$
FOC: $p = 3\lambda C^2 \qquad w = 2000\lambda L \qquad \Rightarrow \qquad \frac{p}{w} = \frac{3C^2}{2000L}$
RC: $1000L^2 = C^3 \qquad \Rightarrow \qquad L = \left(\frac{C}{10}\right)^{3/2} \qquad C^2 = \frac{p}{w}\frac{2000L}{3} = \frac{p}{w}\frac{2000}{3}\left(\frac{C}{10}\right)^{3/2}$
$C = \left(\frac{p}{w}\frac{2000}{3}\right)^2\left(\frac{1}{10}\right)^3 = \boxed{\frac{4000}{9}\frac{p^2}{w^2} = C^S} \qquad \Rightarrow \qquad \boxed{L^D = \left(\frac{20}{3}\frac{p}{w}\right)^3}$
$\pi(p, w) = pC - wL = p\frac{4000}{9}\frac{p^2}{w^2} - w\left(\frac{20}{3}\frac{p}{w}\right)^3 = \frac{4000}{27}\frac{p^3}{w^2}$
b) $\max\{3\ln C + 2\ln(54 - L)\,|wL + \pi(p, w) - pC \geq 0\}$
$\pounds = 3\ln C + 2\ln(54 - L) + \mu(wL + \pi(p, w) - pC)$
FOC: $\frac{3}{C} = \mu p \qquad \frac{2}{54-L} = \mu w \qquad \frac{p}{w} = \frac{3}{C}\frac{54-L}{2}$
BC: $L + \frac{4000}{27}\frac{p^3}{w^3} = \frac{p}{w}C \qquad \Rightarrow \qquad \frac{p}{w} = \frac{3}{2}\frac{54 + \frac{4000}{27}\frac{p^3}{w^3} - \frac{p}{w}C}{C} = \frac{3}{2}\left(54 + \frac{4000}{27}\frac{p^3}{w^3}\right)\frac{1}{C} - \frac{3}{2}\frac{p}{w}$
$\boxed{C^D = \frac{3}{5}\left(54 + \frac{4000}{27}\frac{p^3}{w^3}\right)\frac{w}{p}} \qquad \boxed{L^S} = \frac{p}{w}C - \frac{4000}{27}\frac{p^3}{w^3} = \boxed{\frac{162}{5} - \frac{2}{5}\frac{4000}{27}\frac{p^3}{w^3}}$
c) Equilibirum: $L^S = \left(\frac{20}{3}\frac{p}{w}\right)^3 = L^D = \frac{162}{5} - \frac{2}{5}\frac{4000}{27}\frac{p^3}{w^3}$
$\left(\frac{20}{3}\right)^3\left(\frac{p}{w}\right)^3 = \frac{162}{5} - \frac{2}{5}\frac{4000}{27}\left(\frac{p}{w}\right)^3 \qquad \boxed{\frac{p}{w} = \frac{9}{20} \qquad L = 27 \qquad C = 90}$
c*) Pareto-Optimal is WE (unique):
$\max\{3\ln C + 2\ln(54 - L)\,|1000L^2 - C^3 \geq 0, L \geq 0\}$
$\pounds = 3\ln 10L^{2/3} + 2\ln(54 - L)$
FOC: $\frac{2}{L} = \frac{2}{54-L} \qquad L = 27 \qquad C = 10(27)^{2/3} = 90 \qquad \frac{p}{w} = \sqrt{\frac{90*9}{4000}} = \frac{9}{20}$