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**Model**

$$\max U = E_1 \left\{ \sum_{t=1}^{\infty} \beta^{t-1} u(C_t, L_t) \right\}, \quad 0 < \beta < 1$$

$$\text{where } u(C, L) = \frac{(C^\varphi(1-L)^{1-\varphi})^{1-\rho}-1}{1-\rho}, \quad \rho > 0, \quad 0 < \varphi < 1$$

$$\text{s.t. } K_{t+1} = K_t(1-\delta) + Y_t - C_t, \quad t = \overline{1\infty}, \quad K_1 = \bar{K}$$

$$A_t = A_{t-1}^\lambda \exp(e_t), \quad t = \overline{1\infty}, \quad 0 < \lambda < 1, \quad A_0 = \bar{A}$$

$$Y_t = A_t K_t^\alpha [(1+g)^t L_t]^{1-\alpha}, \quad 0 < \alpha < 1$$

$$\text{Detrend: } k_t = \frac{K_t}{(1+g)^t}, \quad y_t = \frac{Y_t}{(1+g)^t}, \quad c_t = \frac{C_t}{(1+g)^t}$$

$$(1+g)k_{t+1} = k_t(1-\delta) + y_t - c_t \quad y_t = A_t k_t^\alpha L_t^{1-\alpha} \quad A_t = A_{t-1}^\lambda \exp(e_t)$$

$$0 = E_1 \left\{ \beta \frac{(1-\delta) + \alpha \frac{y_{t+1}}{k_{t+1}}}{(1+g)^{1-\varphi(1-\rho)}} c_{t+1}^{\varphi(1-\rho)-1} \left[ \frac{y_{t+1}}{c_{t+1}} \frac{\varphi(1-\alpha)}{(1-\varphi)} + 1 \right]^{(\varphi-1)(1-\rho)} - c_t^{\varphi(1-\rho)-1} \left[ \frac{y_t}{c_t} \frac{\varphi(1-\alpha)}{(1-\varphi)} + 1 \right]^{(\varphi-1)(1-\rho)} \right\}$$

$$u_t = (C_t^\varphi(1-L_t)^{1-\varphi})^{1-\rho} = (c_t^\varphi(1-L_t)^{1-\varphi})^{1-\rho} \times (1+g)^{t\varphi(1-\rho)} \quad 1-L_t = \frac{1}{\frac{y_t}{c_t} \frac{\varphi(1-\alpha)}{(1-\varphi)} + 1}$$

**Steady-state:**

$$A = 1 \quad \frac{y}{k} = \frac{1}{\alpha} \left[ \frac{1}{\beta} (1+g)^{1-\varphi(1-\rho)} - (1-\delta) \right] = \theta$$

$$c/y = 1 - (\delta + g)k/y = 1 - \frac{\delta+g}{\theta} = \varkappa \quad L = 1 - \left( \frac{1}{\varkappa} \frac{\varphi(1-\alpha)}{(1-\varphi)} + 1 \right)^{-1}$$

$$k = L/\theta^{1-\alpha}, \quad y = \theta k, \quad c = \varkappa y$$

**Equations of the model:**

$$k_t(1-\delta) + y_t - c_t = (1+g)k_{t+1} \quad y_t = A_t k_t^\alpha L_t^{1-\alpha} \quad A_t = A_{t-1}^\lambda \exp(e_t) \quad \frac{1-\varphi}{\varphi(1-\alpha)} \frac{L_t}{1-L_t} \frac{c_t}{y_t} = 1$$

$$E_1 \left\{ \beta \frac{(1-\delta) + \alpha \frac{y_{t+1}}{k_{t+1}}}{(1+g)^{1-\varphi(1-\rho)}} c_{t+1}^{\varphi(1-\rho)-1} [1-L_{t+1}]^{(1-\varphi)(1-\rho)} - c_t^{\varphi(1-\rho)-1} [1-L_t]^{(1-\varphi)(1-\rho)} \right\} = 0$$

**Linearize** with respect to  $\{k_t, y_t, c_t, L_t, A_t\}$  using the following rule:

$$\bar{k}(1-\delta) \ln \frac{k_t}{\bar{k}} + \bar{y} \ln \frac{y_t}{\bar{y}} - \bar{c} \ln \frac{c_t}{\bar{c}} = (1+g)\bar{k} \ln \frac{k_{t+1}}{\bar{k}} \quad \ln \frac{A_t}{\bar{A}} = \lambda \ln \frac{A_{t-1}}{\bar{A}} + e_t$$

$$\ln \frac{y_t}{\bar{y}} = \ln \frac{A_t}{\bar{A}} + \alpha \ln \frac{k_t}{\bar{k}} + (1-\alpha) \ln \frac{L_t}{\bar{L}} \quad \ln \frac{y_t}{\bar{y}} = \ln \frac{c_t}{\bar{c}} + \frac{1}{1-L} \ln \frac{L_t}{\bar{L}}$$

$$E_t \left\{ \frac{1}{\frac{1-\delta}{\alpha} \frac{\bar{k}}{\bar{y}} + 1} \left[ \ln \frac{y_{t+1}}{\bar{y}} - \ln \frac{k_{t+1}}{\bar{k}} \right] + [\varphi(1-\rho) - 1] \ln \frac{c_{t+1}}{\bar{c}} - (1-\varphi)(1-\rho) \frac{L}{1-L} \ln \frac{L_{t+1}}{\bar{L}} \right\} =$$

$$[\varphi(1-\rho) - 1] \ln \frac{c_t}{\bar{c}} - \frac{L}{1-L} (1-\varphi)(1-\rho) \ln \frac{L_t}{\bar{L}} + v_{t+1}$$

$$\text{Define more parameters } \phi = \frac{1}{\frac{1-\delta}{\alpha} \frac{\bar{k}}{\bar{y}} + 1}, \omega = \varphi(1-\rho) - 1, \pi = (1-\varphi)(1-\rho) \frac{\bar{L}}{1-L}$$

**Linearized system:**

$$\phi [E_t \hat{y}_{t+1} - \hat{k}_{t+1}] + \omega [E_t \hat{c}_{t+1} - \hat{c}_t] - \pi [E_t \hat{L}_{t+1} - \hat{L}_t] = 0$$

$$\hat{y}_t - (1-\alpha)\hat{L}_t - \hat{a}_t = \alpha \hat{k}_t \quad \hat{y}_t - \hat{c}_t - \frac{1}{1-L} \hat{L}_t = 0 \quad \hat{a}_t = \lambda \hat{a}_{t-1} + e_t$$

$$-\bar{y} \hat{y}_t + (1+g)\bar{k} \hat{k}_{t+1} + \bar{c} \hat{c}_t = \bar{k}(1-\delta) \hat{k}_t$$

$$\begin{bmatrix} 0 & -\phi & -\omega & \pi & 0 & \phi & \omega & -\pi \\ -\bar{y} & (1+g)\bar{k} & \bar{c} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -(1-\alpha) & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & \frac{-1}{1-L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ k_{t+1} \\ c_t \\ L_t \\ a_t \\ E_t y_{t+1} \\ E_t c_{t+1} \\ E_t L_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{k}(1-\delta) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ k_t \\ c_{t-1} \\ L_{t-1} \\ a_{t-1} \\ E_{t-1} y_t \\ E_{t-1} c_t \\ E_{t-1} L_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} e_t + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_t \\ w_t \\ u_t \end{bmatrix}$$

(f) Gensys: 
$$\begin{bmatrix} y_t \\ k_{t+1} \\ c_t \\ L_t \\ a_t \end{bmatrix} = G \begin{bmatrix} y_{t-1} \\ k_t \\ c_{t-1} \\ L_{t-1} \\ a_{t-1} \end{bmatrix} + C e_t \quad \text{Var}[e_t] = \sigma_a - \quad \text{State-space equation}$$

$$\begin{bmatrix} y_t \\ k_{t+1} \\ c_t \\ L_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ k_{t+1} \\ c_t \\ L_t \\ a_t \end{bmatrix} + w_t \quad \text{Var}[w_t] = \text{diag}\{\sigma_y, \sigma_k, \sigma_c, \sigma_l\} - \quad \text{measurement}$$

$\Rightarrow$  Kalman filter  $\Rightarrow$  Metropolis-Hastings

Bayes estimates

	$\beta$	$\gamma$	$\rho$	$\alpha$	$\phi$	$\lambda$	$\delta$	$\sigma_y$	$\sigma_k$	$\sigma_c$	$\sigma_l$	$\sigma_a$
prior med	0.960	1.8e-3	3.00	0.33	0.17	0.65	0.10	0.05	1e-10	1e-4	1.5e-4	7e-5
prior stdev	4e-3	7e-4	0.206	0.021	0.06	0.12	0.04	0.03	6e-11	6e-5	9e-5	4e-5
post med	0.962	1.6e-3	2.95	0.32	0.19	0.66	0.15	0.11	6.7e-11	4.7e-5	4.8e-5	1.2e-4
post stdev	9e-4	1e-4	0.027	0.005	0.01	0.02	0.03	0.03	2e-11	2e-5	5e-5	2e-5

Reject ratio: 33%.



