November 2, 2006

Model

$\max U = E_1\left\{\sum_{t=1}^{\infty}\beta^{t-1}u(C_t,L_t)\right\}, \qquad 0<\beta<1$

where $u(C,L)=\frac{\left(C^{\varphi}(1-L)^{1-\varphi}\right)^{1-\rho}-1}{1-\rho}, \qquad \rho>0, \qquad 0<\varphi<1$

s.t. $K_{t+1}=K_t(1-\delta)+Y_t-C_t, \qquad t=\overline{1\infty}, \qquad K_1=\bar{K}$

$A_t=A_{t-1}^{\lambda}\exp(e_t), \qquad t=\overline{1\infty}, \qquad 0<\lambda<1, \qquad A_0=\bar{A}$

$Y_t=A_tK_t^{\alpha}\left[(1+g)^tL_t\right]^{1-\alpha}, \qquad 0<\alpha<1$

Detrend: $k_t=\frac{K_t}{(1+g)^t}, \qquad y_t=\frac{Y_t}{(1+g)^t}, \qquad c_t=\frac{C_t}{(1+g)^t}$

$(1+g)k_{t+1}=k_t(1-\delta)+y_t-c_t \qquad y_t=A_tk_t^{\alpha}L_t^{1-\alpha} \qquad A_t=A_{t-1}^{\lambda}\exp(e_t)$

$0=E_1\{\beta\frac{(1-\delta)+\alpha\frac{y_{t+1}}{k_{t+1}}}{(1+g)^{1-\varphi(1-\rho)}}c_{t+1}^{\varphi(1-\rho)-1}[\frac{y_{t+1}}{c_{t+1}}\frac{\varphi(1-\alpha)}{(1-\varphi)}+1]^{(\varphi-1)(1-\rho)}-c_t^{\varphi(1-\rho)-1}[\frac{y_t}{c_t}\frac{\varphi(1-\alpha)}{(1-\varphi)}+1]^{(\varphi-1)(1-\rho)}\}$

$u_t=\left(C_t^{\varphi}(1-L_t)^{1-\varphi}\right)^{1-\rho}=\left(c_t^{\varphi}(1-L_t)^{1-\varphi}\right)^{1-\rho}\times(1+g)^{t\varphi(1-\rho)} \qquad 1-L_t=\frac{1}{\frac{y_t}{c_t}\frac{\varphi(1-\alpha)}{(1-\varphi)}+1}$

Steady-state:

$A=1 \qquad \frac{y}{k}=\frac{1}{\alpha}[\frac{1}{\beta}(1+g)^{1-\varphi(1-\rho)}-(1-\delta)]=\theta$

$c/y=1-(\delta+g)k/y=1-\frac{\delta+g}{\theta}=\varkappa \qquad L=1-(\frac{1}{\varkappa}\frac{\varphi(1-\alpha)}{(1-\varphi)}+1)^{-1}$

$k=L/\theta^{\frac{1}{1-\alpha}}, \qquad y=\theta k, \qquad c=\varkappa y$

Equations of the model:

$k_t(1-\delta)+y_t-c_t=(1+g)k_{t+1} \qquad y_t=A_tk_t^{\alpha}L_t^{1-\alpha} \qquad A_t=A_{t-1}^{\lambda}\exp(e_t) \qquad \frac{1-\varphi}{\varphi(1-\alpha)}\frac{L_t}{1-L_t}\frac{c_t}{y_t}=1$

$E_1\{\beta\frac{(1-\delta)+\alpha\frac{y_{t+1}}{k_{t+1}}}{(1+g)^{1-\varphi(1-\rho)}}c_{t+1}^{\varphi(1-\rho)-1}[1-L_{t+1}]^{(1-\varphi)(1-\rho)}-c_t^{\varphi(1-\rho)-1}[1-L_t]^{(1-\varphi)(1-\rho)}\}=0$

Linearize with respect to $\{k_t,y_t,c_t,L_t,A_t\}$ using the following rule:

$\overline{k}(1-\delta)\ln\frac{k_t}{\overline{k}}+\overline{y}\ln\frac{y_t}{\overline{y}}-\overline{c}\ln\frac{c_t}{\overline{c}}=(1+g)\overline{k}\ln\frac{k_{t+1}}{\overline{k}} \qquad \ln\frac{A_t}{\overline{A}}=\lambda\ln\frac{A_{t-1}}{\overline{A}}+e_t$

$\ln\frac{y_t}{\overline{y}}=\ln\frac{A_t}{\overline{A}}+\alpha\ln\frac{k_t}{\overline{k}}+(1-\alpha)\ln\frac{L_t}{\overline{L}} \qquad \ln\frac{y_t}{\overline{y}}=\ln\frac{c_t}{\overline{c}}+\frac{1}{1-\overline{L}}\ln\frac{L_t}{\overline{L}}$

$E_t\{\frac{1}{\frac{1-\delta}{\alpha}\frac{k}{y}+1}[\ln\frac{y_{t+1}}{\overline{y}}-\ln\frac{k_{t+1}}{\overline{k}}]+[\varphi(1-\rho)-1]\ln\frac{c_{t+1}}{\overline{c}}-(1-\varphi)(1-\rho)\frac{L}{1-L}\ln\frac{L_{t+1}}{\overline{L}}\}=$

$[\varphi(1-\rho)-1]\ln\frac{c_t}{\overline{c}}-\frac{L}{1-L}(1-\varphi)(1-\rho)\ln\frac{L_t}{\overline{L}}+v_{t+1}$

Define more parameters $\phi=\frac{1}{\frac{1-\delta}{\alpha}\frac{\overline{k}}{\overline{y}}+1},\omega=\varphi(1-\rho)-1,\pi=(1-\varphi)(1-\rho)\frac{\overline{L}}{1-\overline{L}}$

Linearized system:

$\phi[E_t\hat{y}_{t+1}-\hat{k}_{t+1}]+\omega[E_t\hat{c}_{t+1}-\hat{c}_t]-\pi[E_t\hat{L}_{t+1}-\hat{L}_t]\}=0$

$\hat{y}_t-(1-\alpha)\hat{L}_t-\hat{a}_t=\alpha\hat{k}_t \qquad \hat{y}_t-\hat{c}_t-\frac{1}{1-\overline{L}}\hat{L}_t=0 \qquad \hat{a}_t=\lambda\hat{a}_{t-1}+e_t$

$-\overline{y}\hat{y}_t+(1+g)\overline{k}\hat{k}_{t+1}+\overline{c}\hat{c}_t=\overline{k}(1-\delta)\hat{k}_t$

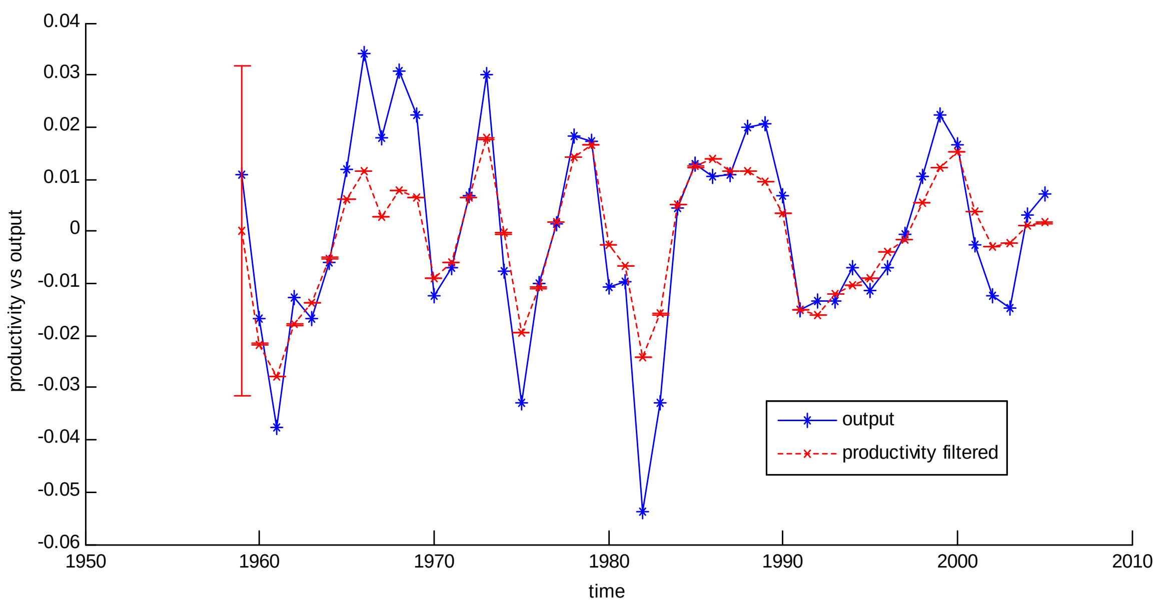
$$
\begin{bmatrix}
0 & -\phi & -\omega & \pi & 0 & \phi & \omega & -\pi \\
-\overline{y} & (1+g)\overline{k} & \overline{c} & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -(1-\alpha) & -1 & 0 & 0 & 0 \\
1 & 0 & -1 & \frac{-1}{1-\overline{L}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
y_t \\ k_{t+1} \\ c_t \\ L_t \\ a_t \\ E_t y_{t+1} \\ E_t c_{t+1} \\ E_t L_{t+1}
\end{bmatrix}
=
$$

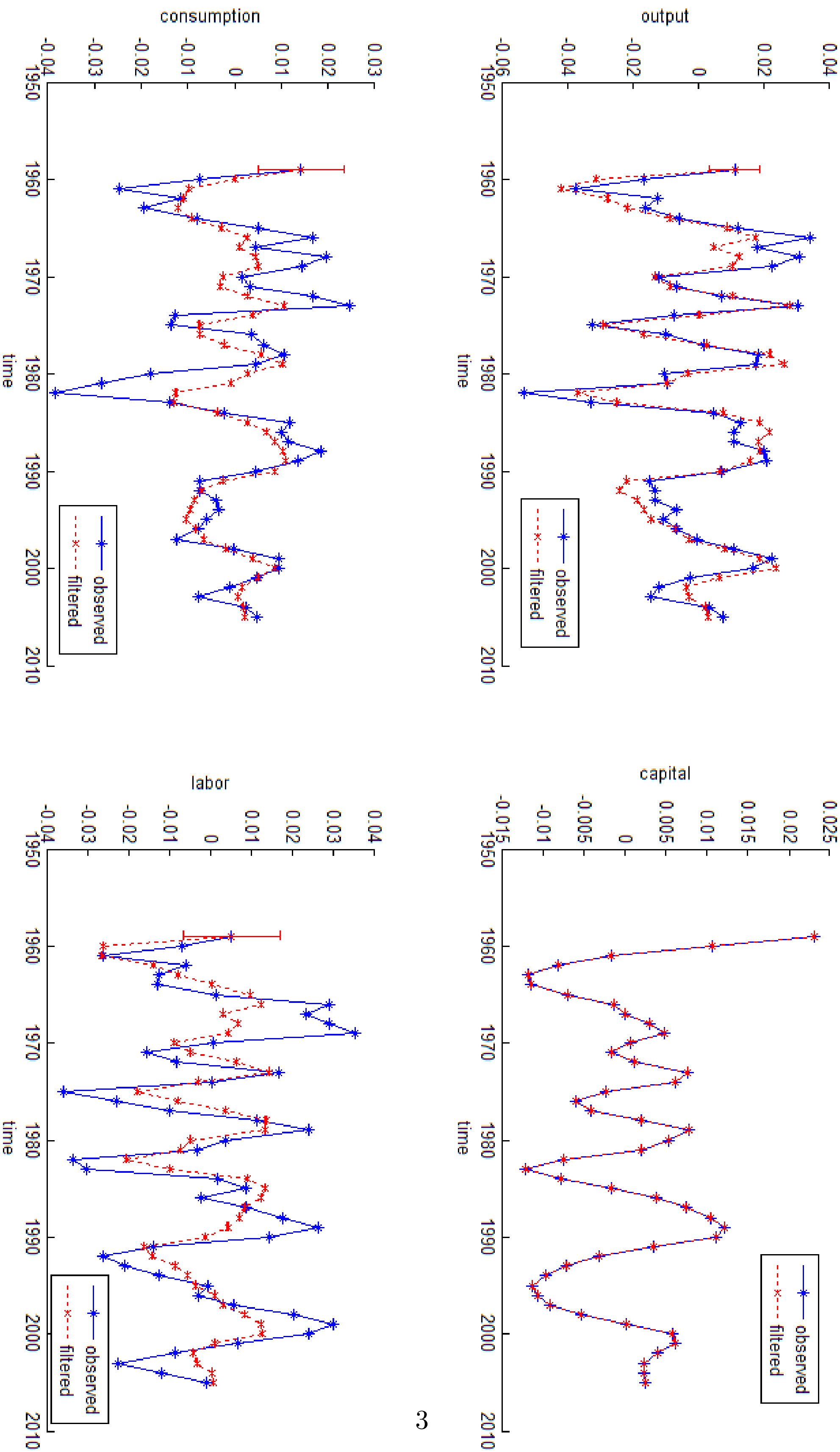
$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \overline{k}(1-\delta) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
y_{t-1} \\ k_t \\ c_{t-1} \\ L_{t-1} \\ a_{t-1} \\ E_{t-1} y_t \\ E_{t-1} c_t \\ E_{t-1} L_t
\end{bmatrix}
+
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0
\end{bmatrix}
e_t +
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
v_t \\ w_t \\ u_t
\end{bmatrix}
$$

(f) Calibrated: $\beta = 0.96$, $\rho = 3$, $\alpha = 1/3$.
Starting values: $g = 0.02, \varphi = 1/2, \lambda = 0.95, \delta = 0.1$

Gensys gives the solution:
$$
\begin{bmatrix}
y_t \\ k_{t+1} \\ c_t \\ L_t \\ a_t
\end{bmatrix}
= G
\begin{bmatrix}
k_t \\ a_{t-1}
\end{bmatrix}
+ C e_t
$$

I use Kalman Filter to calibrate the model: $g = 0.002, \varphi = 0.19, \lambda = 0.65, \delta = 0.07$

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capital
consumption
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time
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1950
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