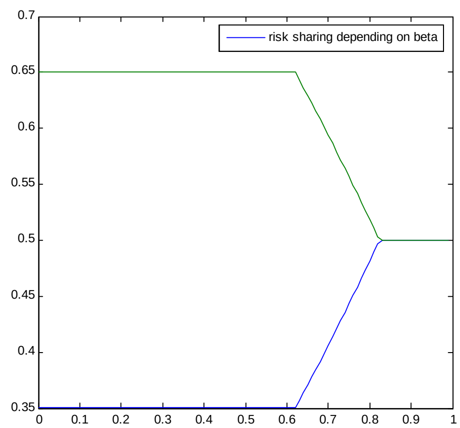
Spring 2007, Economics 221C, Homework 1 Anton Cheremukhin

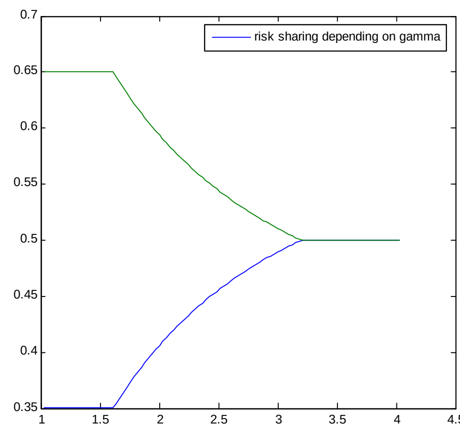
The *limited commitment problem* is set as follows:

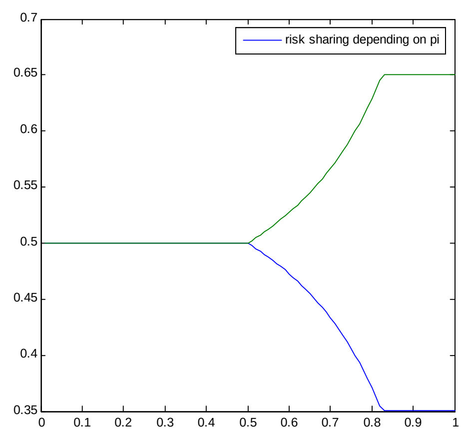
There are two agents, whose states ('lo' and 'hi') are perfectly anticorrelated. Agents have power preferences are given by $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. They discount time by $\beta = 0.65$ and are risk-averse with parameter $\gamma = 2$. Their incomes in states 'lo' and 'hi' are $y = [0.35; 0.65]$ respectively. Let's denote w the promised utility and c the corresponding consumption levels. Persistence is given by the probability of remaining in the same state $p = 0.75$. Utility in autarky is defined as: $U_{aut} = (I - \beta P)^{-1} u(y)$. Perfect risk sharing is possible, if $U_{aut}(hi) < U(1/2)$. Then the solution to the limited commitment problem satisfies the system of equations:

$$w = u(c) + \beta P w \qquad [1,1]\, c = 1 \qquad w(hi) = \max(U_{aut}(hi), U(1/2))$$

Because autarky is always a solution, we use a numerical procedure using perfect risk sharing as the starting point. The solution, corresponding to the given values is $c = [0.3702, 0.6294]$. If we iterate over parameters, we can get that:







More generally the solution to the limited commitment problem has to satisfy the system of equations:

$$w = u(c) + \beta P w \qquad Mc = 1 \qquad w(hi_i) = \max(U_{aut}(hi_i), U(1/2))$$

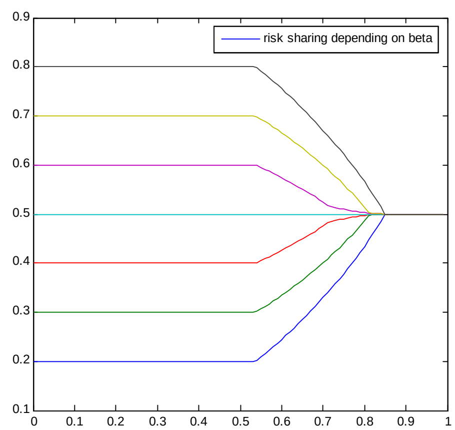
For example, imagine we have 7 states with endowments $y = [0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8]$, and the transition matrix is defined as:

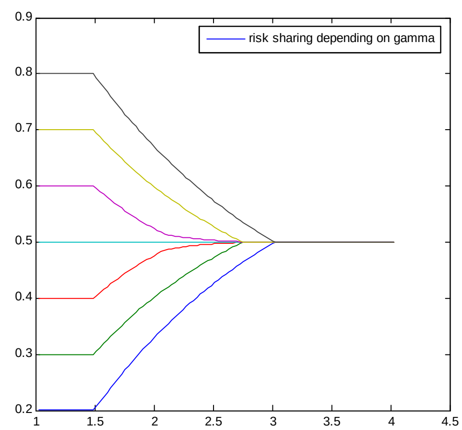
$$P = \begin{bmatrix} p & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} \\ \frac{1-p}{6} & p & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} \\ \frac{1-p}{6} & \frac{1-p}{6} & p & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} \\ \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & p & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} \\ \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & p & \frac{1-p}{6} & \frac{1-p}{6} \\ \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & p & \frac{1-p}{6} \\ \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & \frac{1-p}{6} & p \end{bmatrix}$$

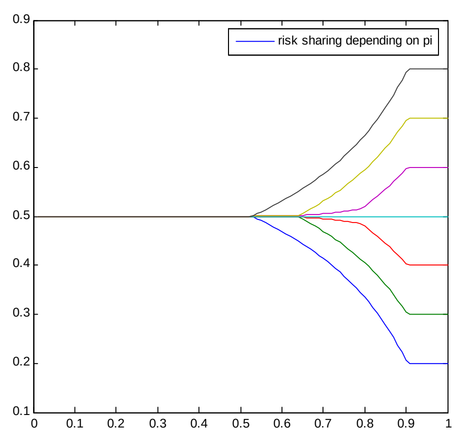
We need to solve the following system of equations: $\quad w = u(c) + \beta P w$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \end{bmatrix} c = 1 \qquad w\left(\begin{bmatrix} 0.8 \\ 0.7 \\ 0.6 \end{bmatrix}\right) = \max\left\{U_{aut}\left(\begin{bmatrix} 0.8 \\ 0.7 \\ 0.6 \end{bmatrix}\right), U\left(\tfrac{1}{2}\right)\right\}$$

If we iterate over parameters, we can get that:







Perfect risk-sharing is possible if the agents are patient enough, or risk-averse enough, or the shocks are not very persistent.