Economics 281A, Homework 4 Anton Cheremukhin

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Factor Mobility and Technological Differences

Consider a Heckscher-Ohlin Model with two goods and two factors. Assume that international prices are determined in global markets and fixed. Assume that one factor is internationally mobile and the other is fixed in the country of origin. Assume that there are two countries A and B and A is has more advanced technology than B in one sector and the same in the other. Fully describe the international equilibrium for both cases: the difference is in the sector using the mobile/immobile factor more intensively.

Let world prices be p_1 and p_2. Let's assume that capital is mobile and labor is immobile.

$K_{A1}+K_{A2}+K_{B1}+K_{B2}=K \qquad L_{A1}+L_{A2}=L_A \qquad L_{B1}+L_{B2}=L_B$

Let both technologies be Cobb-Doublas and the advanced country just have a higher TFP in one sector: $q_{A1}=AK_{A1}^{\alpha}L_{A1}^{1-\alpha} \qquad q_{B1}=K_{B1}^{\alpha}L_{B1}^{1-\alpha} \qquad q_{A2}=K_{A2}^{\beta}L_{A2}^{1-\beta} \qquad q_{B2}=K_{B2}^{\beta}L_{B2}^{1-\beta}$

A1: $p_1AK_{A1}^{\alpha}L_{A1}^{1-\alpha}-w_AL_{A1}-rK_{A1}\to\max$ FOCs:$(1-\alpha)\,p_1AK_{A1}^{\alpha}L_{A1}^{-\alpha}=w_A \quad \alpha p_1AK_{A1}^{\alpha-1}L_{A1}^{1-\alpha}=r$

B1: $p_1K_{B1}^{\alpha}L_{B1}^{1-\alpha}-w_BL_{B1}-rK_{B1}\to\max$ FOCs:$(1-\alpha)\,p_1K_{B1}^{\alpha}L_{B1}^{-\alpha}=w_B \quad \alpha p_1K_{B1}^{\alpha-1}L_{B1}^{1-\alpha}=r$

A2: $p_2K_{A2}^{\beta}L_{A2}^{1-\beta}-w_AL_{A2}-rK_{A2}\to\max$ FOCs:$(1-\beta)\,p_2K_{A2}^{\beta}L_{A2}^{-\beta}=w_A \quad \beta p_2K_{A2}^{\beta-1}L_{A2}^{1-\beta}=r$

B2: $p_2K_{B2}^{\beta}L_{B2}^{1-\beta}-w_BL_{B2}-rK_{B2}\to\max$ FOCs:$(1-\beta)\,p_2K_{B2}^{\beta}L_{B2}^{-\beta}=w_B \quad \beta p_2K_{B2}^{\beta-1}L_{B2}^{1-\beta}=r$

Therefore, $\frac{K_{A1}}{L_{A1}}=\left[\frac{w_A}{(1-\alpha)p_1}\right]^{1/\alpha}=\left[\frac{\alpha p_1}{Ar}\right]^{1/(1-\alpha)} \qquad \frac{K_{B1}}{L_{B1}}=\left[\frac{w_B}{(1-\alpha)p_1}\right]^{1/\alpha}=\left[\frac{\alpha p_1}{r}\right]^{1/(1-\alpha)}$

$\frac{K_{A2}}{L_{A2}}=\left[\frac{w_A}{(1-\beta)p_2}\right]^{1/\beta}=\left[\frac{\beta p_2}{r}\right]^{1/(1-\beta)} \qquad \frac{K_{B2}}{L_{B2}}=\left[\frac{w_B}{(1-\beta)p_2}\right]^{1/\beta}=\left[\frac{\beta p_2}{r}\right]^{1/(1-\beta)}$

$K_{A1}+K_{A2}+K_{B1}+K_{B2}=K \qquad L_{A1}+L_{A2}=L_A \qquad L_{B1}+L_{B2}=L_B$

Hence (under assumption of an interior solution for everybody),

$\left[\frac{\beta p_2}{r}\right]^{1/(1-\beta)}=\frac{K_{A2}}{L_{A2}}=\frac{K_{B2}}{L_{B2}} \qquad w_B=(1-\beta)\,p_2\left[\frac{\beta p_2}{r}\right]^{\beta/(1-\beta)}=w_A$

$\left[\frac{\alpha p_1}{Ar}\right]^{1/(1-\alpha)}=\left[\frac{w_A}{(1-\alpha)p_1}\right]^{1/\alpha}=\left[\frac{w_B}{(1-\alpha)p_1}\right]^{1/\alpha}=\left[\frac{\alpha p_1}{r}\right]^{1/(1-\alpha)} \qquad \Rightarrow \qquad A=1 \quad$ (contradiction).

We assume A>1 (country A is more productive in good 1). Then, there are 4 cases.

Case 1: $\alpha>\beta$ (technological difference is in the sector using mobile factor more intensively)

Capital flows from country B to country A. Here there are 2 subcases:

1a) $L_{A2}=0$ - country A specializes in good 1, country B mixes (enough labor in B).

1b) $L_{B1}=0$ - country B specializes in good 2, country A mixes. (not enough labor in B).

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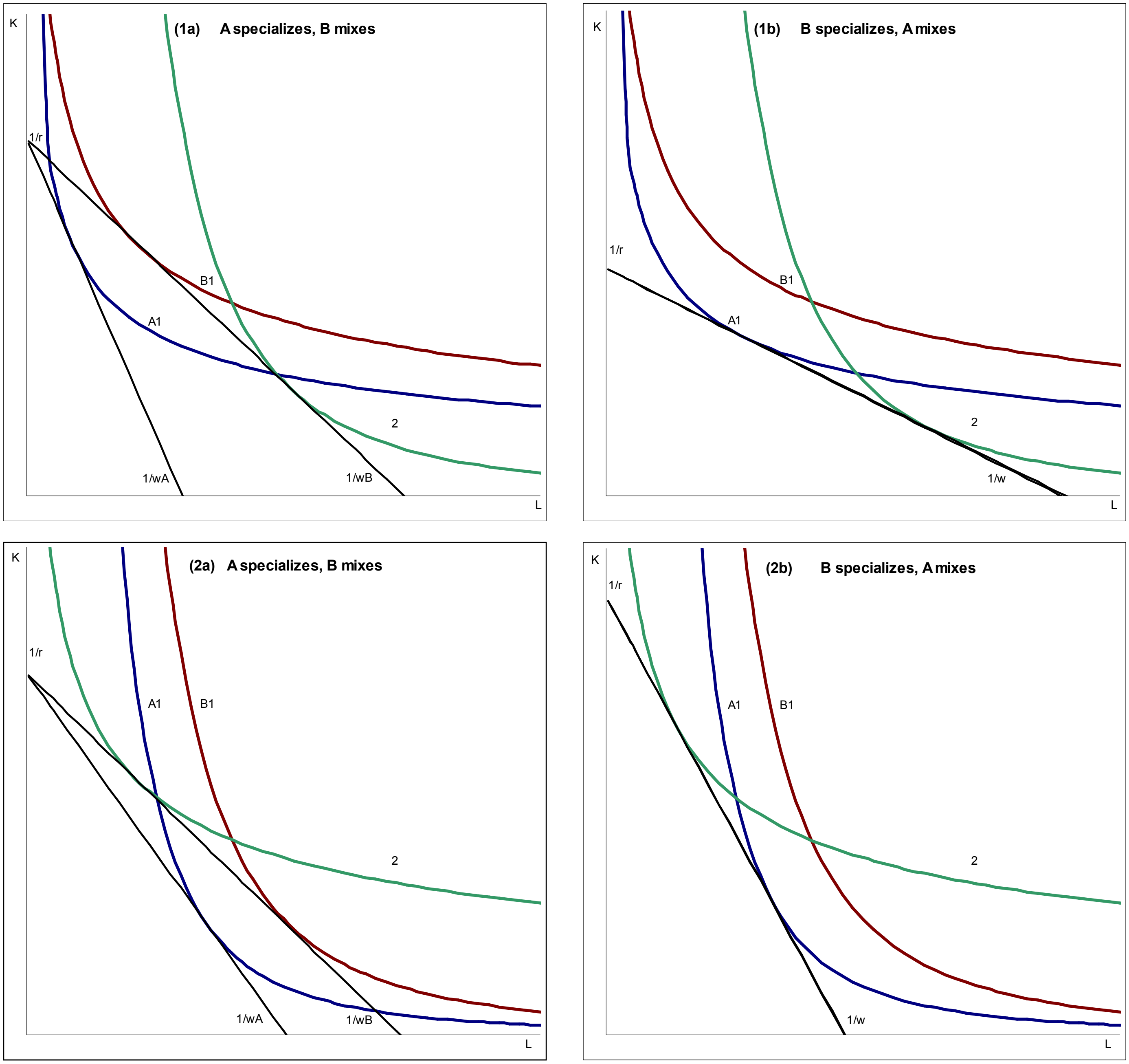
Case 2: $\alpha<\beta$ (technological difference is in the sector using immobile factor more intensively)

Capital flows from country A to country B. Here there are 2 subcases:

2a) $L_{A2}=0$ - country A specializes in good 1, country B mixes. (not too much labor in B).

2b) $L_{B1}=0$ - country B specializes in good 2, country A mixes. (too much labor in B).

The cases are graphically represented in the following pictures.


(1a) A specializes, B mixes
K
1/r
B1
A1
2
1/wA
1/wB
L
(1b) B specializes, A mixes
K
1/r
B1
A1
2
1/w
L
(2a) A specializes, B mixes
K
1/r
A1
B1
2
1/wA
1/wB
L
(2b) B specializes, A mixes
K
1/r
A1
B1
2
1/w
L