

Homework #1  
Due Monday, April 24

You can work in groups of two people. Include a printout of the relevant graphs.

1. In this exercise you will analyze the effects of aggregation of time series in the presence of a common factor. Write a function that, for given values of  $N$ ,  $\sigma_x^2$ , performs the following steps

- Using the random number generator (e.g., in Matlab, the function "randn"), generate  $N$  independent white noise series of length  $T = 500$  (the idiosyncratic components):

$$X_{jt} \sim i.i.d.N(0, \sigma_x^2), \quad j = 1, \dots, N, \quad t = 1, \dots, T$$

- Generate a variable  $Z_t$  (the common factor) as an  $AR(1)$  :

$$Z_t = 0.7Z_{t-1} + \varepsilon_t,$$

with  $\varepsilon_t \sim i.i.d.N(0, 1)$ .

- Define the new series

$$Y_{jt} = Z_t + X_{jt}, \quad j = 1, \dots, N, \quad t = 1, \dots, T.$$

- To verify what the time series properties of the individual disaggregated series  $Y_{jt}$  are, plot the first 10 lags of the sample autocorrelogram of  $Y_{1t}$ , together with approximate confidence intervals.
- Now consider the cross sectional aggregated variable

$$SY_t = \sum_{j=1}^N Y_{jt}$$

- Plot the first 10 lags of the sample autocorrelogram of  $SY_t$ .

Answer the following questions:

- (a) What is the variance of the common factor  $Z_t$ ?
- (b) Experiment with different values of  $N$  (e.g.,  $N = 10, 100, 500$ ) and of the idiosyncratic error variance  $\sigma_x^2$ . Find values of  $N$  and  $\sigma_x^2$  so that the individual series (e.g.,  $Y_{1t}$ ) look near white noise whereas the aggregate  $SY_t$  looks  $AR(1)$ . (The answer is not unique).
- (c) Briefly comment your findings and their implications for time series modelling of aggregated variables.

2. In this exercise we consider US labor productivity and establish whether the series has a structural break. You will essentially replicate the results of Hansen, B. (2001): "The New Econometrics of Structural Change: Dating Breaks in U.S. Labor Productivity", *Journal of Economic Perspectives*, 15, 117-128. Go to FRED II (<http://research.stlouisfed.org/fred2/>) and download monthly Industrial Production Index and Average Weekly Hours: Total Private Industries from 1964:1 onward. Construct a measure of labor productivity by considering the logarithmic growth rate of the ratio Industrial Production/Hours.
- (a) Estimate an AR(1) model for labor productivity and test the hypothesis that the AR coefficient is zero.
  - (b) Use a Breusch-Godfrey test to see whether the residuals from the regression in a.) are serially correlated
  - (c) Use Andrews (1993) test to verify whether there was a structural break in the unconditional mean of labor productivity (consider the interior 5-95% of the sample and use the heteroskedasticity-robust version of the test statistic).
  - (d) If the above test finds evidence of a break, estimate the timing of the break using Bai's (1994) method.