

November 2, 2006

1. Results

MA(2) process: $y_t = \mu + \varepsilon_t + q_1\varepsilon_{t-1} + q_2\varepsilon_{t-2}$ $\varepsilon_t \sim N(0, \sigma^2)$

It has the following state-space representation:

$$\begin{bmatrix} \varepsilon_t \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{t-1} \\ \varepsilon_{t-2} \\ \varepsilon_{t-3} \end{bmatrix} + \begin{bmatrix} v_{1t} \\ v_{2t} \\ v_{3t} \end{bmatrix} \quad \text{Var}[v] = \begin{bmatrix} \sigma^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$y_t - \mu = \begin{bmatrix} 1 & q_1 & q_2 \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \end{bmatrix} + w_t \quad \text{Var}[w_t] = [0]$$

In the code we used functions:

"kalman_filter" - to estimate the likelihood given the parameters

"csminwel" - to find the parameters maximizing the likelihood.

The optimal parameters are: $q_1 = 0.58$ $q_2 = 0.05$ $\mu = 0.91$ $\sigma^2 = 0.82$

Code:

```
clear all; close all; clc; addpath(genpath('C:/Matlab7/toolbox/KalmanAll'))
[y] = textread('MA2.txt','%f'); y=y';
q0=[0.6, 0.2, 0, 1]; H0=0.001*eye(4); crit=0.0001; nit=1000;
[fh,x,gh,H,itct,fcount,retcodeh] = csminwel(@makelike2, q0, H0, [], crit, nit, y);
```

```
function [L] = makelike2(thet,y);
q1=thet(1); q2=thet(2); m=thet(3); sig=thet(4);
F = [0 0 0; 1 0 0; 0 1 0]; H = [1 q1 q2]; Q = diag([sig 0 0]); R = 0;
initx = [y(1)-m;0;0]; initV = 0.00000001*eye(3);
[xfilt, Vfilt, VVfilt, loglik] = kalman_filter(y-m, F, H, Q, R, initx, initV);
L=-loglik;
```