Problem Set 1

Econ 212A: Search and Matching Theory

Pierre-Olivier Weill

Consider the following version of Lucas-Prescott islands model.

Information and Technology. Time is discrete and runs forever. There is a measure-one continuum of island in which competitive firm operate a production function $n \mapsto \theta f(n)$, where $f(n)$ is some increasing, concave function of labor input n, and θ is an island specific productivity shock. It is assumed that, in each island productivity is a two-state Markov chain which can be either low, θ_L, or high, $\theta_H > \theta_L$. An island's productivity remains constant from one period to another with probability $\pi \in (.5, 1)$, and its productivity changes to the other possible value with probability $1 - \pi$. Lastly, it is assumed that productivity is i.i.d across islands.

Preferences. There is a measure-one continuum of infinitely lived and risk-neutral workers, who discount the future at the same rate $\beta \in (0, 1)$. Every period, a worker chooses either: i) to stay in his current island and supply one unit of labor inelastically, ii) to move to some other island, and wait one period to supply labor.

Notations. As in the class presentation, we let x be the labor force of an island, at the beginning of the period after the productivity shock realizes, but before workers decide wether or not to move out of the island. Then, a number $\Delta^- \geq 0$ of workers move out, a number $\Delta^+ \geq 0$ of workers move in. However, the worker who move in cannot work this period, so that the employment level in the island is

$$n = x - \Delta^-,$$

and, at the beginning of next period, the workforce is

$$x' = x - \Delta^- + \Delta^+.$$

1. What is the stationary distribution of productivity across islands?
2. Recall the cutoff rule characterizing labor movements in equilibrium. Argue that, in this two-state example, if in equilibrium workers do move across islands, then an island's labor force has two possible values, $\{x_1, x_2\}$ with $0 < x_1 < x_2$.

3. In a stationary equilibrium with labor movements, construct a matrix Γ with the transition probabilities between states (θ, x), and explain what the employment level is in different states.

4. Let $v(x, \theta)$ be the maximum attainable utility of a worker that starts the period in an island with current labor force x, and current productivity θ. In a stationary equilibrium with labor movements, we observe only four values of the value function $v(\theta, x)$ where $\theta \in \{\theta_L, \theta_H\}$ and $x \in \{x_1, x_2\}$. Argue that the value function takes on the same value for two of these four states.

5. Show that the condition for the existence of a stationary equilibrium with labor movements is

$$\beta(2\pi - 1)\theta_H \; > \; \theta_L \,, \tag{1}$$

and, if this condition is satisfied, an implicit expression for the equilibrium value of x_2 is

$$[\theta_L \, + \, \beta(1 - \pi)\theta_H] \, f'(x_1) \;\; = \;\; \beta\pi\theta_H f'(x_2) \,. \tag{2}$$

6. Verify that the allocation of labor in part d. coincides with a social planner's solution when maximizing the present value of the economy's aggregate production, subject to the moving technology (i.e. that a worker can't work in the first period after he moves).

   Hint: Here are two ways to answer this question:

   (a) One way is to set-up the full blown planner's inter-temporal problem, letting the planner choose the number of household to move in and out of an island as a function of the history of productivity shock. Taking first-order conditions should give you condition (2).

   (b) Another way is to try some variational experiment. That is, starting in some steady state, engineer a small *feasible* move of households across island, calculate the first-order welfare gain. If the steady state distribution is an optimum, then the welfare gain is zero and this should give you condition (2).

   (It is good that you try both!!!)