Economics 203C **Moshe Buchinsky**
Introduction to Econometrics–System Models **Department of Economics**
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Problem Set 5

Due: Monday May 22, 2006

Question 1:

Consider the Generalized Method of Moments (GMM) estimator defined in Lecture Note 10:

$$\widehat{\theta}_n = \arg\min_{\theta\in\Theta} m_n(\theta)' V_n^{-1} m_n(\theta),$$

where

$$V_n \xrightarrow{p} V, \quad \text{a non-stochastic non-singular matrix}$$
$$m_n(\theta) = \frac{1}{n}\sum_{i=1}^{n} \varphi(y_i, x_i; \theta), \quad \text{and}$$
$$E_0[\varphi(y_i, x_i; \theta_0)] = 0.$$

a. Show that $\widehat{\theta}_n$ is a consistent estimator for θ_0.

b. Show that $\widehat{\theta}_n$ is asymptotically normal, that is $\sqrt{n}\left(\widehat{\theta}_n - \theta_0\right) \xrightarrow{D} N(0, V)$.

Note: The proofs should follow the same steps as for the proof of consistency for the MLE. In each step make sure to state the exact assumption(s) needed for the statement(s) made.

Question 2:

Consider the following model:

$$y_i^* = z_i'\gamma + u_i,$$

for $i = 1, \ldots n$, where u_i conditional on z_i has a normal distribution, that is,

$$u_i|z_i \sim N\left(0, \sigma_v^2\right).$$

Define

$$y_i = \begin{cases} 1 & \text{if } y_i^* > 0, \\ 0 & \text{otherwise.} \end{cases}$$

a. Show that $\Pr(y_i = 1|z_i) = \Phi(z_i'\gamma/\sigma_v)$. Explain why γ and σ_v cannot be separately identified. Denote $\theta = \gamma/\sigma_v$, and let θ_0 be the population parameter.

b. Provide the normalized log likelihood function for θ.

c. Provide the first order conditions for $\widehat{\theta}_n$, the estimator for θ_0.

d. Show that the estimator obtained by solving the first order conditions in (c) can be viewed as a Method of Moments estimator.

Question 3:

Consider the linear model given by

$$y_i = x_i'\beta_0 + \varepsilon_i$$
$$E[\varepsilon_i|x_i] = 0,$$

for $i = 1, ...n$, where x_i is a $K \times 1$ vector of regressors, that is, $x_i = (x_{1i}, ..., x_{Ki})'$.

Define the vector

$$z_i = \left(x_i', x_{1i}^2, ..., x_{Ki}^2, x_{1i}x_{2i}, ..., x_{1i}x_{Ki}\right)',$$

and the following moment functions:

$$\begin{aligned} \varphi_1(y_i, x_i; \beta) &= \left(y_i - x_i'\beta\right) x_i, \quad \text{and} \\ \varphi_2(y_i, z_i; \beta) &= \left(y_i - x_i'\beta\right) z_i. \end{aligned}$$

a. Define the population parameter vector β_0.

b. Show that when evaluated at the population value β_0,

$$\begin{aligned} E_0[\varphi_1(y_i, x_i; \beta_0)] &= 0, \quad \text{and} \\ E_0[\varphi_2(y_i, z_i; \beta_0)] &= 0. \end{aligned}$$

c. Define the optimal GMM estimators for β_0 based on $\varphi_1(y_i, x_i; \beta)$ and $\varphi_2(y_i, z_i; \beta)$, say $\widehat{\beta}_n^1$, and $\widehat{\beta}_n^2$, respectively.

d. Provide the asymptotic covariance matrices for $\widehat{\beta}_n^1$ and $\widehat{\beta}_n^2$, from (c).

e. Provide a consistent estimator for the asymptotic covariance matrix of $\widehat{\beta}_n^2$ from (d). Show that the proposed estimator is, in fact, consistent estimator for its population counterpart, using the assumptions put forward in Lecture Note 10.

f. Provide the asymptotic distribution for the GMM estimator based on $\varphi_2(y_i, z_i; \beta)$ when the weight matrix is $V_n = I$.

Question 4:

For this exercise we use the results of question 3 above. In the excel file **ps5q4.xls** you are provided with the data for this exercise. There are six variables in columns 1 through 6 of the file, corresponding to $y, x_1, ..., x_5$, respectively, where $x_{1i} = 1$ for all $i = 1, ..., n$.

a. The estimator in this part is based on $\varphi_1(y_i, x_i; \beta)$ from Question 3.

   1. Provide an estimate for β_0 and the corresponding standard errors, based on $V_n = I$.
   2. Provide the optimal estimate for β_0 and the corresponding standard errors, based on the optimal matrix V_n.
   3. Compare the two estimates obtained in (a.1) and (a.2).

b. Repeat the exercise in (a) for $\varphi_2(y_i, z_i; \beta)$.

c. Compare and discuss the differences and similarities between the optimal estimates (and their corresponding vectors of standard errors) from (a) and (b).

d. Construct a consistent estimate for the optimal weight matrix base on the optimal estimate from (b) and re-estimate the optimal GMM. Discuss the results in comparison with the results obtained in (b).

e. Construct the Wald statistic for testing the hypothesis H_0: $\beta_2^2 + \beta_4^2 = \beta_3^2 + \beta_5^2$ based on the optimal estimates for β_0, from both (a) and (b). Discuss briefly your conclusions?