Economics 203C — **Moshe Buchinsky**
Introduction to Econometrics–System Models — **Department of Economics**
Spring, 2006 — **UCLA**

Problem Set 4

Due: Monday May 8, 2006

Question 1:

Consider the asymptotic distribution of the MLE estimator derived in Lecture Note 8. Explain in detail whether or not

$$\left(\frac{1}{n}\sum_{i=1}^{n}\frac{\partial \ln f(y_i, x_i; \theta_0)}{\partial \theta}\right)\left(\frac{1}{n}\sum_{i=1}^{n}\frac{\partial \ln f(y_i, x_i; \theta_0)}{\partial \theta'}\right)$$

is a consistent estimator for $I(\theta_0)$.

Question 2 (question 3, in Greene, page 522):

Suppose that the joint distribution of the two random variables x and y is

$$f(x, y) = \frac{\theta e^{-(\beta+\theta)y}(\beta y)^x}{x!}, \quad \beta > 0,\ \theta > 0,\ y \geq 0,\ x = 0, 1, 2, ...$$

1. Find the MLE for β and θ and provide their asymptotic joint distribution.

2. Find the MLE for $\theta/(\beta + \theta)$ and its asymptotic distribution.

3. Prove that $f(x)$ is of the form

$$f(x) = \gamma(1 - \gamma)^x, \quad x = 0, 1, 2, ...$$

   and find the MLE estimator for γ and its asymptotic distribution.

4. Prove that $f(y|x)$ is of the form

$$f(y|x) = \frac{\lambda e^{-\lambda y}(\lambda y)^x}{x!}, \quad y \geq 0,\ \lambda > 0.$$

   Show that $f(y|x)$ integrates to 1. Find the MLE for λ and its asymptotic distribution.

5. Prove that

$$f(y) = \theta e^{-\theta y}, \quad y \geq 0,\ \theta > 0.$$

   Find the MLE for θ and its asymptotic distribution.

6. Prove that

$$f(x|y) = \frac{e^{-\beta y}(\beta y)^x}{x!}, \quad \beta > 0, \quad x = 0, 1, 2, ...$$

   Find the MLE for β and its asymptotic distribution.

Question 3:

Consider the binary probit model given by

$$\Pr(y_i|x_i;\theta) = \Phi(x_i'\gamma), \quad i = 1, ...n,$$

where $\Phi(\cdot)$ denote the cdf of a standard normal variable.

1. Define the population parameter vector γ_0.

2. Define the sample log-likelihood function $L(\gamma)$ and the first-order conditions for the MLE, say $\widehat{\gamma}_n$.

3. Find the asymptotic distribution of the MLE, that is, show that

$$\sqrt{n}\left(\widehat{\gamma}_n - \gamma_0\right) \xrightarrow{D} N\left(0, \Lambda_0\right)$$

   and provide the exact formula for Λ_0.

4. Provide two alternative estimator for the asymptotic covariance matrix, one that is based on the Hessian matrix and one that is based on the outer product gradient (OPG). For each of these two estimators, show that it is a consistent estimator for the asymptotic covariance matrix derived in (3). State clearly all the assumptions you make in order to show consistency of the estimators.

Question 4:

Consider the data provided in **ps4q4.xls** provided on the class website. In this data file you are provided with data on y, x_1, x_2, x_3, and x_4, where y_i, $(i = 1, ..., n)$ takes on two possible values 0 (failure) or 1 (success), according to the following model

$$y_i = I(\beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \varepsilon_i > 0),$$

where $\varepsilon_i|x_i \sim N(0,1)$, $x_i = (x_{1i}, x_{2i}, x_{3i}, x_{4i})'$, and $I(\cdot)$ denotes the usual indicator function. The data are also stored in the matlab file **ps4q4.mat** which contains a 500×5 matrix with the variables y, x_1, x_2, x_3, and x_4 in the five columns, respectively.

1. Provide the MLE for $\beta = (\beta_1, \beta_2, \beta_3, \beta_4)'$, say $\widehat{\beta}_n$.

2. Provide a consistent estimate for the asymptotic covariance matrix of the estimator derived in (1). Provide also the sample standard errors for $\widehat{\beta}_n$.

3. Test the null hypothesis H_0: $\beta_2 + \beta_3 = 0$ against H_1: Not H_0.

4. Provide the elasticities for x_2, x_3, and x_4 when the probabilities are evaluated at the sample average of x_2, x_3, and x_4.

5. Consider now the function

$$h(\beta) = \frac{\beta_1\beta_2}{\beta_3^2}.$$

   Provide an estimate for $h(\beta)$ and an estimate for it standard error using the delta method.