Economics 203C | **Moshe Buchinsky**
Introduction to Econometrics–System Models | **Department of Economics**
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Problem Set 3

Due: Monday, May 1, 2006

Question 1:

This question is based on a statement made in Lecture Note 5. Consider the IV estimator of the form

$$\widehat{\beta}_{IV} = \left(\widehat{\Pi}' Z' X\right)^{-1} \widehat{\Pi}' Z' y.$$

1. Suppose that $\widehat{\Pi} \xrightarrow{p} \Pi_0$ as $n \to \infty$. Provide the asymptotic distribution for $\widehat{\beta}_{IV}$ and the asymptotic covariance matrix, say Λ_0.
2. Suppose now that $\widehat{\Pi} \xrightarrow{p} V_0^{-1}\Sigma_{zx}$, where V_0 and Σ_{zx} are defined in the lecture note. Provide the asymptotic distribution for $\widehat{\beta}_{IV}$ and the asymptotic covariance matrix, say Λ^*.
3. Show that the asymptotic covariance matrix in (1) is at least as large as the asymptotic covariance matrix in (2) in matrix sense, that is, show that $\Lambda_0 - \Lambda^*$ is a positive definite matrix.

Question 2:

Consider the SUR model given by

$$y_{ij} = x_{ij}'\beta_j + \varepsilon_{ij}, \quad i = 1, ..., n;\ j = 1, 2, 3,$$

with

$$\varepsilon_i \sim \text{i.i.d.}(0, \Sigma),$$

where $\varepsilon_i = (\varepsilon_{i1}, \varepsilon_{i2}, \varepsilon_{i3})'$. Also, assume that $E\left[x_{ij}\varepsilon_i\right] = 0$ for $j = 1, 2, 3$. In the EXCEL file **ps3q2.xls** you are provided with the data for this problem set. There are 13 columns and 500 rows (i.e., observations) in this file. The first three columns contain the data on y_1, y_2, and y_3, respectively. The rest of the columns contain the data on $x_1, ..., x_{10}$. In addition there is a Matlab file called **ps3q2.mat** that contains the same data in `y1, y2, y3,` and `X`.

Suppose now that the first equation has only x_1, x_2, x_3, and x_4 as explanatory variables, the second equation has x_5, x_6, x_7, and x_8 as explanatory variables, while the third equation has x_1, x_2, x_5, x_6, x_9 and x_{10} as explanatory variables.

1. Provide the OLS estimate for the parameter vectors β_j, $j = 1, 2, 3$.
2. Provide a consistent estimate for Σ based on the regression estimates from (1).
3. Provide the standard error estimates for each of the three vectors of coefficient estimates obtained in (1).
4. Provide the GLS estimates of the parameter vectors β_j, $j = 1, 2, 3$, using the estimate for Σ obtained in (2).

5. Provide the standard error estimates for each of the three vectors of coefficient estimates obtained in (4).

6. Briefly discuss the results obtained in (1) and (4) for the point estimates for β_j, $j = 1, 2, 3$.

7. Briefly discuss the results obtained in (3) and (5) for the standard error estimates for $\widehat{\beta}_j$, $j = 1, 2, 3$.

8. Suppose now that it was claimed that the coefficients on x_1 and x_2 in the first and third regression should be the same, and that the coefficients on x_5 and x_6 in the second and third equations should be the same. Provide an estimate for all three parameter vectors that incorporate these constraints.

9. Test that hypothesis of the claims made in (8).

Question 3 (question 5, in Greene, page 423):

Consider the following model:

$$\begin{aligned} y_{1i} &= \gamma_1 y_{2i} + \beta_{11} x_{1i} + \varepsilon_{1i}, \\ y_{2i} &= \gamma_2 y_{1i} + \beta_{22} x_{2i} + \beta_{32} x_{3i} + \varepsilon_{2i}. \end{aligned}$$

All variables are measured as deviations from their means. The sample of 25 observations produces the following matrix of sums of squares and cross products:

$$\begin{array}{c} \\ y_1 \\ y_2 \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccccc} 20 & 6 & 4 & 3 & 5 \\ 6 & 10 & 3 & 6 & 7 \\ 4 & 3 & 5 & 2 & 3 \\ 3 & 6 & 2 & 10 & 8 \\ 5 & 7 & 3 & 8 & 15 \end{array}\right] \end{array}$$

1. Estimate the two equations by OLS.

2. Estimate the parameters of the two equations by 2SLS. Also, estimate the asymptotic covariance matrix of the 2SLS estimates.

3. Obtain the LIML estimates of the parameters of the first equation.

4. Estimate the two equations by 3SLS.

5. Estimate the reduced-form coefficient matrix by OLS, and indirectly by using the structural estimates from part (2).