Economics 203C **Moshe Buchinsky**
Introduction to Econometrics–System Models **Department of Economics**
Spring, 2006 **UCLA**

Problem Set 1

Due: Monday April 17, 2006

Question 1:

Consider the simple consumption model given by

$$c_i = \beta_1 + \beta_2 y_i^* + u_i, \quad u_i \sim \text{i.i.d. } (0, \sigma^2),$$

where c_i is the logarithm of consumption by household i, and y_i^* is the permanent income of that household, which is not observed. Instead, we only observe the current household income y_i

$$y_i = y_i^* + v_i,$$

where v_i ~i.i.d. $(0, \omega^2)$, is assume to uncorrelated with y_i^* and u_i. Consider now the regression

$$c_i = \beta_1 + \beta_2 y_i + \varepsilon_i.$$

Under some assumption it is reasonable to believe that the true parameter β_{20} is positive.

a. Show that y_i is negatively correlated with ε_i.

b. Using the result in (a), compute probability limit of the OLS estimator for β_{20}, say $\widehat{\beta}_2$, from the second regression. show that this probability limit is less than β_{20}.

Question 2:

Suppose that the data generating process that simultaneously determine x and y is given by

$$\begin{aligned} y &= x\beta_0 + \sigma_u u, \\ x &= w\pi_0 + \sigma_v v, \end{aligned}$$

where y, x, u and v are all $n \times 1$ vectors, u and v have standard normal distribution and $E(u_i v_i) = \rho$. Suppose also that there exist an instrument for x, say w. Assume that $w'w = 1$.

a. Show that $E(u_i|v_i) = \rho v_i$, so we can write $u_i = \rho v_i + \varepsilon_i$, where $E(\varepsilon_i|v_i) = 0$.

b. Define the instrumental variable for β_0, say $\widehat{\beta}_{IV}$.

c. Show that for the estimator defined in (b)

$$\widehat{\beta}_{IV} - \beta_0 = \frac{\sigma_u w'(\rho v + \varepsilon)}{\pi_0 + \sigma_v w'v},$$

where $\varepsilon = (\varepsilon_1, ..., \varepsilon_n)'$.

d. Show that if $\sigma_v = 0$, then the estimator defined in (b) is an unbiased estimator for β_0. Interpret this result.

Question 3:

Consider the regression model given by

$$y_i = x_i'\beta_0 + u_i, \quad i = 1, ..., n,$$

where x_i is a $k \times 1$ vector of regressors and β_0 is a $k \times 1$ vector of unknown parameters. Assume that $u_i \sim$ i.i.d.$(0, \sigma_u^2)$, but $E(x_i u_i) \neq 0$. Let z_i be an $l \times 1$ vector, with $l > k$.

a. What would constitute z_i as an instrument for x_i? Discuss briefly.

b. Suppose we define

$$\hat{\beta}_{IV} = \left(X'ZA^{-1}Z'X\right)^{-1} X'ZA^{-1}Z'y$$

where X is a $n \times k$ matrix with rows given by x_i', y is an $n \times 1$ vector of the stacked y_i, Z is an $n \times l$ matrix with rows given by z_i', and A is an $n \times n$ non-stochastic, non-singular, matrix. Show that $\hat{\beta}_{IV}$ is a consistent estimator for β_0.

c. Show that, under some regularity conditions,

$$\sqrt{n}\left(\hat{\beta}_{IV} - \beta_0\right) \xrightarrow{D} N(0, \Lambda),$$

and provide Λ.