Econ203C Homework #6 Anton Cheremukhin

June 1, 2006

Exercise 1 *GMM and the Three Tests*

The hardest part is to get a stable estimate for both the unrestricted and the restricted version. We do this in six steps.

1) We find unrestricted $\beta^{(1)}$ using and identity weight matrix and "fminsearch".
2) Given that, we compute the optimal weight matrix $V^{(2)}$ and find the corresponding $\beta^{(2)}$.
3) We Repeat step 2 and obtain $V^{(3)}$ and the corresponding $\beta^{(3)}$.
4) We Compute A and $m_n\left(\beta^{(3)}\right)$.
5) We find restricted $\beta^{(5)}$ using $V^{(3)}$ as the weight and "fmincon".
6) We compute $m_n\left(\beta^{(5)}\right)$ and $V^{(5)}$.

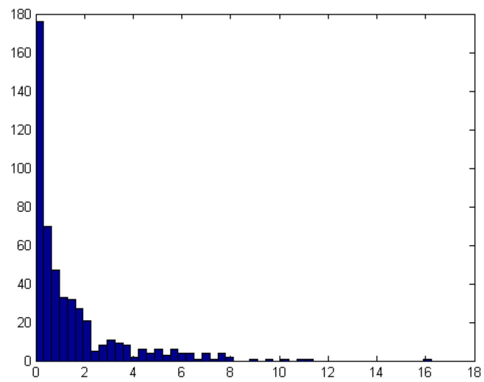
After that we calculate the three tests: $W = nr'\left(R\left(A\left(V^{(3)}\right)^{-1}A'\right)^{-1}R'\right)r$,

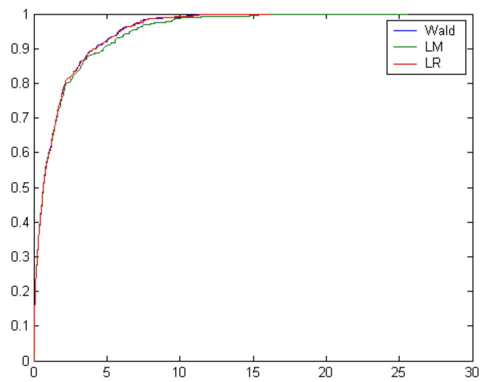
$LM = nm_n\left(\beta^{(5)}\right)'\left(V^{(3)}\right)^{-1}A'\left(A\left(V^{(5)}\right)^{-1}A'\right)^{-1}A\left(V^{(3)}\right)^{-1}m_n\left(\beta^{(5)}\right)$,

$LR = n\left(m_n\left(\beta^{(5)}\right)'\left(V^{(3)}\right)^{-1}m_n\left(\beta^{(5)}\right) - m_n\left(\beta^{(3)}\right)'\left(V^{(3)}\right)^{-1}m_n\left(\beta^{(3)}\right)\right)$,

where $r = \beta_2\beta_3 + \beta_4$, $R = [0, \beta_3, \beta_2, 1]$.

The reason why we use $V^{(3)}$ is because it is the weight matrix, we used when finding the minimum - the one we regard as 'optimal'. Besides, otherwise we often get negative values for the LR test. The whole procedure is very sensitive to $V^{(3)}$, that's why we repeated the second step, to be sure we are getting a good estimate of the 'optimal' weight matrix.





The histograms are almost identical: one of them is depicted on the left. The graph on the right compares the three empirical c.d.f.'s of the tests. They are almost indistinguishable. The probabilities of type I errors for the 95% level are all around 11-12%. The means differ a little, which can be clearly seen in the graph.

	Wald	LM	LR
mean	1.47	1.62	1.47
median	0.67	0.67	0.66
P(type I error)	0.109	0.118	0.108

Exercise 2 *NLLS as an Example of GMM*

$y_i = g(x_i, \theta_0) + u_i \qquad E(u_i|x_i) = 0$

1) $E\left[(y_i - g(x_i, \theta))^2\right] \to \min_\theta$ use law of iterated expectations:

FOC: $E\left[2\frac{\partial}{\partial\theta} g(x_i, \theta)(y_i - g(x_i, \theta))\right] = E\left[2\frac{\partial}{\partial\theta} g(x_i, \theta) E\left[(y_i - g(x_i, \theta))|x_i\right]\right] = 0$

since $E(u_i|x_i) = 0 \Leftrightarrow E(y_i|x_i) = g(x_i, \theta_0)$, $\qquad E\left[2\frac{\partial}{\partial\theta} g(x_i, \theta)(g(x_i, \theta_0) - g(x_i, \theta))\right] = 0$

If $g(.)$ is strictly monotone in θ, this expression is satisfied iff $\theta = \theta_0$.

2) The usual conditions for existence are continuity of $g(.)$ and compactness of Θ. For uniqueness we need to add: $\theta \neq \theta_0 \Rightarrow g(x, \theta) \neq g(x, \theta_0)$. Strict monotonicity in θ is one possibility.

3) $\widehat{\theta}_n = \arg\min_\theta m_n(\theta)' (V_n)^{-1} m_n(\theta)$

$m_n(\theta) = \frac{1}{n}\Sigma\varphi(y_i, x_i, \theta) \qquad \varphi(y_i, x_i, \theta) = z(y_i - g(x_i, \theta))$

Some simple ideas for instruments z_{ji} include: $1, x_{1i}, ..., x_{Ki}$.

4) Some additional simple ideas for instruments z_{ji} include: $x_{1i}^2, ..., x_{Ki}^2$.

We can use higher powers, if needed. The justification to that would be a Taylor expansion of $\frac{\partial}{\partial\theta} g(x_i, \theta)$ around θ_0. These are all instruments since $E\left[f(x_i)(y_i - g(x_i, \theta))\right] = 0$ only in θ_0.

5) An optimal GMM estimator uses an optimal weight matrix:

$\widehat{V}_n = \frac{1}{n}\Sigma\varphi\left(y_i, x_i, z, \widehat{\theta}'\right)\varphi\left(y_i, x_i, z, \widehat{\theta}'\right)'$.

Can use any consistent $\widehat{\theta}'$, e.g. $\widehat{\theta}' = \arg\min_\theta m_n(\theta)' m_n(\theta)$.

6) Asymptotic distribution: $\sqrt{n}\left(\widehat{\theta}_n - \theta_0\right) \xrightarrow{d} N(0, \Lambda(\theta_0))$, where

$A(\theta_0) = E_0\left[\frac{\partial\varphi(y,x,z,\theta_0)}{\partial\theta}\right] \qquad , V(\theta_0) = E_0\left[\varphi(y, x, z, \theta_0)\varphi(y, x, z, \theta_0)'\right]$

$\Lambda(\beta_0) = \left(A(\theta_0) V(\theta_0)^{-1} A(\theta_0)'\right)^{-1}$

A consistent estimator for $A(\theta_0)$ is $\widehat{A} = \sum \frac{\partial\varphi\left(y_i, x_i, z_i, \widehat{\theta}'\right)}{n\partial\beta}$

A consistent estimator for $V(\theta_0)$ is $\widehat{V}_n = \frac{1}{n}\sum z_i z_i'\left(y_i - x_i'\widehat{\theta}'\right)^2$ - non-singular, p.s.d.

Proof is exactly the same as always.

7) $H_0: \theta_1\theta_2...\theta_K = 1 \qquad H_1: \theta_1\theta_2...\theta_K \neq 1$

$r(\theta) = \theta_1\theta_2...\theta_K - 1 \qquad R(\theta) = [\theta_2...\theta_K, \theta_1\theta_3...\theta_K, ..., \theta_1\theta_2...\theta_{K-1}]$

$\widetilde{\theta}_n = \arg\min_\theta m_n(\theta)'\left(\widetilde{V}_n\right)^{-1} m_n(\theta)$ s.t. $r(\theta) = 0 \qquad \widetilde{A} = \sum \frac{\partial\varphi\left(y_i, x_i, z_i, \widetilde{\theta}'\right)}{n\partial\beta}$

$\widetilde{V}_n = \frac{1}{n}\Sigma\varphi\left(y_i, x_i, z_j, \widetilde{\theta}'\right)\varphi\left(y_i, x_i, z_i, \widetilde{\theta}'\right)'$, $\qquad \widetilde{\theta}' = \arg\min_\theta m_n(\theta)' m_n(\theta)$ s.t. $r(\theta) = 0$

$$W = nr\left(\widehat{\theta}\right)'\left(R\left(\widehat{\theta}\right)\left(\widehat{A}\left(\widehat{V}\right)^{-1}\widehat{A}'\right)^{-1} R\left(\widehat{\theta}\right)'\right) r\left(\widehat{\theta}\right),$$

$$LM = nm\left(\widetilde{\theta}\right)'\left(\widetilde{V}\right)^{-1}\widetilde{A}'\left(\widetilde{A}\left(\widetilde{V}\right)^{-1}\widetilde{A}'\right)^{-1}\widetilde{A}\left(\widetilde{V}\right)^{-1} m\left(\widetilde{\theta}\right),$$

$$LR = n\left(m\left(\widetilde{\theta}\right)'\left(\widetilde{V}\right)^{-1} m\left(\widetilde{\theta}\right) - m\left(\widehat{\theta}\right)'\left(\widehat{V}\right)^{-1} m\left(\widehat{\theta}\right)\right) \qquad \text{All distributed as } \chi^2(1).$$