PROBLEM SET 4

PROBLEMS DUE FRIDAY 2/24

Do the empirical exercise from Hayashi's chapter 1 as well as the following problems:

Problem 1:
This problem asks you to perform a Monte Carlo exercise to study the finite sample properties of the OLS and GLS estimators, the estimators of their standard errors and of associated $t-$ statistics..
(a) Generate 1000 samples of size n for (y_i, x_i) from the (homoskedastic) model

$$y_i = x_i\beta + \varepsilon_i$$

where $x_i \sim N(0,1), \varepsilon_i \sim N(0,\sigma^2)$ and $\beta = 1$. Calculate the OLS estimator, its standard errors, and its associated $t-$ statistics for testing the null hypothesis $H_0 : \beta = 1$ both under the assumption of homoskedasticity and under the assumption of (unknown form) of heteroskedasticity. Then examine the empirical size of your two $t-$ tests at the 5% and 10% nominal levels by calculating the percentage of times the $t-$ statistics reject the null using the appropriate 5 and 10% critical values. Do this for $n = 100$, 400, 1600, and for $\sigma^2 = 0.5$, 1.0 and 2.0.
(b) Repeat the exercise above for $\varepsilon_i \sim N(0,\sigma_i^2)$ where $\sigma_i^2 = x_i^2/c$ for $c = 1, 2$ and 4.
(c) For the two sets of Monte Carlo experiments of parts (a) and (b) compute the (infeasible) and the feasible GLS estimators under the assumption that $\sigma_i^2 = x_i^2/c$. Calculate the mean and median bias, root mean squared and median absolute error and graph the empirical distribution of the GLS and FGLS estimators.
(d) For the two sets of Monte Carlo experiments of parts (a) and (b) compute the standard errors and the associated $t-$ statistics for testing the null hypothesis $H_0 : \beta = 1$ for both the infeasible and the feasible GLS estimators. Then examine the empirical size of the $t-$ tests at the 5% and 10% nominal levels by calculating the percentage of times the $t-$ statistics reject the null using the appropriate 5 and 10% critical values.
Comment on your findings, especially on the relative performance of OLS and GLS (and FGLS) under both homoskedasticity and heteroskedasticity and their relative efficiency.

Problem 2:
Two samples of 50 observations each produce the following moment matrices. (In each case, the matrix X contains the unit vector and one variable.)

$$X'X : \quad \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix} \qquad \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix}$$

$$X'Y : \quad \begin{bmatrix} 300 \\ 2000 \end{bmatrix} \qquad \begin{bmatrix} 300 \\ 2200 \end{bmatrix}$$

$$Y'Y : \quad 2100 \qquad 2800$$

(a) Compute the least squares regression coefficients and the residual variances $\hat{\sigma}^2$ for each data set. Compute the R^2 for each regression.
(b) Compute the OLS estimate of the coefficient vector assuming that the coefficients and disturbance variance are the same in the two regressions. Also compute the estimate of the asymptotic covariance matrix of the estimate.
(c) Test the hypothesis that the variances in the two regressions are the same without assuming that the coefficients are the same in the two regressions.
(d) Compute the two-step FGLS estimator of the coefficients in the regressions, assuming that the constant and slope are the same in both regressions. Compute the estimate of the covariance matrix and compare it with the result of part b.

Problem 3:
Suppose we have n i.i.d. observations on (Y_i, X_i) from the $K-$ variate linear regression model

$$Y_i = X_i\beta + \varepsilon_i$$

where $E(\varepsilon_i|x_i) = 0$ and $Var(\varepsilon_i|x_i) = \sigma^2$. Under what conditions is the following estimator of σ^2

$$\hat{\sigma}^2 = \frac{1}{n-K}\sum_{i=1}^{n}\left(Y_i - X_i\hat{\beta}_{OLS}\right)^2$$

consistent and asymptotically normal? Derive the asymptotic variance and provide a consistent estimator for it.

Problem 4:
Suppose we have n i.i.d. observations on (Y_i, X_i) from the $K-$ variate linear regression model

$$Y_i = X_i\beta + \varepsilon_i$$

where X_i is fixed across repeated samples and $E(\varepsilon_i) = 0$ and $Var(\varepsilon_i) = \sigma^2$ and ε_i is independent of ε_j for $i \neq j$. Derive the asymptotic properties of $\hat{\beta}_{OLS}$. Make sure to state all necessary assumptions and theorems.

Problem 5:
A groupwise heteroskedastic regression has

$$Y_i = X_i\beta + \varepsilon_i$$

where

$$E(\varepsilon_i|X_i) = 0$$

The n independent observations are grouped into G groups each n_g observations. The slope vector β is the same in all groups but within group g,

$$Var(\varepsilon_i|x_i) = \sigma_g^2$$

(a) Derive the GLS, FGLS, and ML estimator of β.
(b) Derive the Likelihood Ratio test for group-wise heteroskedasticity and state its asymptotic distribution.