Econ203B Homework #4 Anton Cheremukhin

February 27, 2006

Hayashi Problem

(a) User cost of capital: $(r+\delta)P_K$

Nerlove definition: rP_K $\Rightarrow$ Depreciation ommited.

(b) Unrestricted model:

logTC	const	logQ	logP1	logP2	logP3
beta	-3.5265	0.7204	0.4363	-0.2199	0.4265
st.er.	1.7744	0.0175	0.2910	0.3394	0.1004
t-stat	-1.9875	41.2445	1.4992	-0.6478	4.2495
p-val	0.0488	0.0000	0.1361	0.5182	0.0000

R^2=0.9260 P-val(CRS)=0.45 CRS not rejected.

(c) Restricted model:

logTC/P3	const	logQ	logP1/P3	logP2/P3
beta	-4.6908	0.7207	0.5929	-0.0074
st.er.	0.8849	0.0174	0.2046	0.1907
t-stat	-5.3011	41.3340	2.8983	-0.0387
p-val	0.0000	0.0000	0.0044	0.9692

R^2=0.9316 P-val(CRS)=0.00 CRS rejected.

(d) Divide into 5 subgroups:

subgroup	1	2	3	4	5
beta 2	0.4003	0.6582	0.9383	0.9120	1.0444
Error Var.	0.3554	0.0588	0.0392	0.0145	0.0226
Ret.ToScale	2.4982	1.5194	1.0658	1.0964	0.9575
SSR	8.8852	1.4691	0.98016	0.36359	0.5644

As output increases betas increase and error variances fall. Returns to scale fall from 2.5 to 1. SSR falls.

(e) Restrict variance to be the same: coefficients coincide.

SSR=12.2624 equal to sum of SSRs.

Proof for 2 groups: $X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}$ $\quad (X'X)^{-1} = \begin{bmatrix} (X'_1X_1)^{-1} & 0 \\ 0 & (X'_2X_2)^{-1} \end{bmatrix}$

$X'Y = \begin{bmatrix} X_1'Y_1 \\ X'_2Y_2 \end{bmatrix}$ $\quad (X'X)^{-1}X'Y = \begin{bmatrix} (X'_1X_1)^{-1}X_1'Y_1 \\ (X'_2X_2)^{-1}X'_2Y_2 \end{bmatrix}$ $\Rightarrow$ beta and SSR the same.

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(f) F-stat of hypothesis: all betas don't differ across groups 5.9747

P-value of hypothesis: all betas don't differ across groups 1.4511e-009. Rejected.

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(g) F-stat of hypothesis: betas 3 and 4 don't differ across groups 0.40123

P-value of hypothesis: betas 3 and 4 don't differ across groups 0.91808. Not rejected.

(h) First step: estimate model assuming homoscedastic errors:

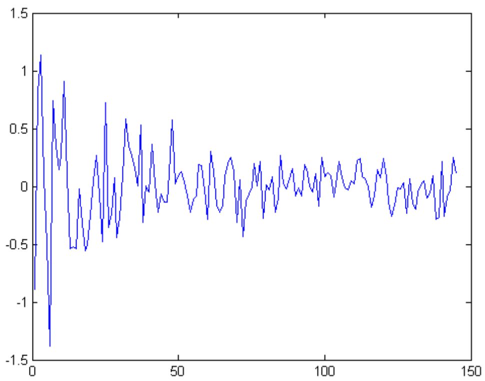
logTC/P3	const	logQ	logQ*logQ	logP1/P3	logP2/P3
beta	-3.7646	0.1525	0.0505	0.4806	0.0742
st.er.	0.7017	0.0619	0.0054	0.1611	0.1500
t-stat	-5.3648	2.4660	9.4178	2.9837	0.4944
p-val	0.0000	0.0149	0.0000	0.0034	0.6218

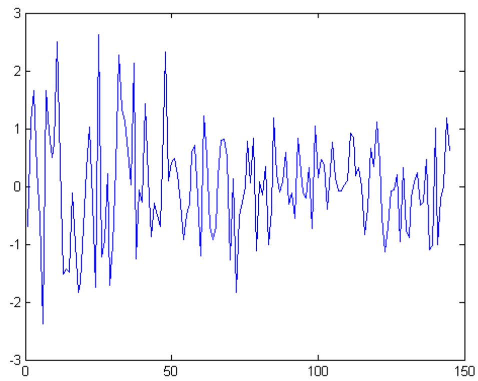
Second step: regress squared errors on inverse of Q
Third step: Take square roots of fitted values and divide the initial regression by them:

e^2	const	1/Q
beta	0.0565	2.1377
st.er.	0.0163	0.2603
t-stat	3.4693	8.2136
p-val	0.0000	0.0000

$\frac{logTC/P3}{\sqrt{e^2}}$	$\frac{const}{\sqrt{e^2}}$	$\frac{logQ}{\sqrt{e^2}}$	$\frac{logQ*logQ}{\sqrt{e^2}}$	$\frac{logP1/P3}{\sqrt{e^2}}$	$\frac{logP2/P3}{\sqrt{e^2}}$
beta	-4.1256	0.2177	0.0457	0.4662	0.1335
st.er.	0.5791	0.0819	0.0062	0.1214	0.1126
t-stat	-7.1242	2.6600	7.3302	3.8393	1.1854
p-val	0.0000	0.0087	0.0000	0.0002	0.2379

Errors before and after correction for heteroscedasticity: a large improvement.





Exercise 1

In this problem we perform 1000 Monte-Carlo experiments with two models: with homoscedastic and heteroscedastic errors.

Model 1: $y_i = x_i + \varepsilon_i, \qquad x_i \sim N(0,1), \qquad \varepsilon_i \sim N(0,\sigma^2) \qquad i = \overline{1,n}$

9 cases are required: $n = \{100, 400, 1600\} \times \sigma^2 = \{0.5, 1.0, 2.0\}$

Model 2: $y_i = x_i + \varepsilon_i, \qquad x_i \sim N(0,1), \qquad \varepsilon_i \sim N(0,\sigma_i^2) \qquad \sigma_i^2 = x_i^2/c \qquad i = \overline{1,n}$

9 cases are required: $n = \{100, 400, 1600\} \times c = \{1, 2, 4\}$

In each case we need to compute 4 estimators and their parameters:

1. $\hat{\beta}_{OLS}$, t-statistic of $H_0 : \beta = 1$ using OLS standard errors, percentage of rejects.
2. $\hat{\beta}_{OLS}$, t-statistic of $H_0 : \beta = 1$ using White standard errors, percentage of rejects.
3. $\hat{\beta}_{GLS}$, t-statistic of $H_0 : \beta = 1$ assuming full info, percentage of rejects, mean bias, median bias, root mean square error, median absolute error and plot of distribution of $\hat{\beta}_{GLS}$.
4. $\hat{\beta}_{FGLS}$, t-statistic of $H_0 : \beta = 1$ using two-step procedure, percentage of rejects, mean bias, median bias, root mean square error, median absolute error and plot of distribution of $\hat{\beta}_{FGLS}$.

According to the results under model 1 all the estimates are unbiased and consistent. Their performance is almost identical. Under model 2, OLS is biased and inconsistent, White is biased but consistent. GLS and FGLS estimates are Cauchi which has no mean, so they don't converge. However, the empirical size of GLS is correct. Bias of OLS generally decreases with the increase in sample size.

Exercise 2

$$1)\ X'X = \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix} \qquad X'Y = \begin{bmatrix} 300 \\ 2000 \end{bmatrix} \qquad Y'Y = 2100 \qquad n = 50$$

$$2)\ X'X = \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix} \qquad X'Y = \begin{bmatrix} 300 \\ 2200 \end{bmatrix} \qquad Y'Y = 2800 \qquad n = 50$$

(a) 1. $\hat{\beta}_{OLS} = \begin{bmatrix} 2 \\ \frac{2}{3} \end{bmatrix} \qquad \hat{\sigma}^2 = \frac{Y'Y - \hat{\beta}'X'X\hat{\beta}}{n-k} = \frac{2100 - \frac{5800}{3}}{50-2} = \frac{125}{36} \qquad R^2 = \frac{4}{9}$

2. $\hat{\beta}_{OLS} = \begin{bmatrix} -2 \\ \frac{4}{3} \end{bmatrix} \qquad \hat{\sigma}^2 = \frac{2800 - \frac{7000}{3}}{50-2} = \frac{175}{18} \qquad R^2 = 1 - \frac{Y'Y - \hat{\beta}'X'X\hat{\beta}}{Y'Y - n(X_1'Y/n)^2} = \frac{8}{15}$

(b) $\hat{\beta}_{OLS} = \left(2 * \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix}\right)^{-1} \left(\begin{bmatrix} 300 \\ 2000 \end{bmatrix} + \begin{bmatrix} 300 \\ 2200 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\hat{\sigma}^2 = \frac{Y'Y - \hat{\beta}'X'X\hat{\beta}}{n-k} = \frac{1}{100-2}\left(2800 + 2100 - 2 * \begin{bmatrix} 0 \\ 1 \end{bmatrix}' \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \frac{50}{7}$

$\widehat{V\left(\hat{\beta}_{OLS}\right)} = \frac{50}{7}\left(2 * \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix}\right)^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{14} \\ -\frac{1}{14} & \frac{1}{84} \end{bmatrix}$

(c) $\frac{RSS_2}{RSS_1} = \frac{e_2'e_2}{e_1'e_1} = \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2} \sim F(n-k, n-k) \qquad \frac{175}{18} / \frac{125}{36} = \frac{14}{5} = 2.8 > F_{0.05,48,48} = 1.58$

The hypothesis of equal variances is rejected.

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(d) Compute RSSs for the two regressions beta OLS from b:

$\hat{\sigma}_1^2 = \frac{1}{50-2}\left(2100 - 2\begin{bmatrix} 0 \\ 1 \end{bmatrix}' \begin{bmatrix} 300 \\ 2000 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}' \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \frac{25}{6}$

$\hat{\sigma}_2^2 = \frac{1}{50-2}\left(2800 - 2\begin{bmatrix} 0 \\ 1 \end{bmatrix}' \begin{bmatrix} 300 \\ 2200 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}' \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \frac{125}{12}$

$\Omega = \begin{bmatrix} 25/6 & 0 \\ 0 & 125/12 \end{bmatrix} \qquad \hat{\beta}_{FGLS} = (X'\Omega^{-1}X)^{-1}(X'\Omega^{-1}Y) =$

$\left(\left(\frac{6}{25} + \frac{12}{125}\right) \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix}\right)^{-1} \left(\frac{6}{25}\begin{bmatrix} 300 \\ 2000 \end{bmatrix} + \frac{12}{125}\begin{bmatrix} 300 \\ 2200 \end{bmatrix}\right) = \begin{bmatrix} \frac{6}{7} \\ \frac{6}{7} \end{bmatrix}$

$V\left(\hat{\beta}_{FGLS}\right) = (X'\Omega^{-1}X)^{-1} = \left(\left(\frac{6}{25} + \frac{12}{125}\right) \begin{bmatrix} 50 & 300 \\ 300 & 2100 \end{bmatrix}\right)^{-1} = \begin{bmatrix} \frac{5}{12} & -\frac{5}{84} \\ -\frac{5}{84} & \frac{5}{504} \end{bmatrix}$

$V\left(\hat{\beta}_{FGLS}\right)^{-1} - V\left(\hat{\beta}_{OLS}\right)^{-1} = \begin{bmatrix} \frac{84}{5} & \frac{504}{5} \\ \frac{504}{5} & \frac{3528}{5} \end{bmatrix} - \begin{bmatrix} 14 & 84 \\ 84 & 588 \end{bmatrix} = \begin{bmatrix} \frac{14}{5} & \frac{84}{5} \\ \frac{84}{5} & \frac{588}{5} \end{bmatrix} > 0$

Hence, $V\left(\hat{\beta}_{FGLS}\right) < V\left(\hat{\beta}_{OLS}\right)$

Exercise 3

$\hat{\sigma}^2 = \frac{e'e}{n-k} = \frac{\varepsilon'M\varepsilon}{n-k} = \frac{n}{n-k}\left[\frac{\Sigma\varepsilon'\varepsilon}{n} - \left(\frac{\Sigma x'\varepsilon}{n}\right)'\left(\frac{\Sigma x'x}{n}\right)^{-1}\left(\frac{\Sigma x'\varepsilon}{n}\right)\right] \xrightarrow{p} \sigma^2$ (by slutsky) consistent

$\frac{n}{n-k} \xrightarrow{p} 1 \qquad$ by l.l.n. $\frac{\Sigma\varepsilon'\varepsilon}{n} \xrightarrow{p} \sigma^2 \qquad \frac{\Sigma x'\varepsilon}{n} \xrightarrow{p} 0 \qquad$ by l.l.n.+contin $\left(\frac{\Sigma x'x}{n}\right)^{-1} \xrightarrow{p} E(x'x)^{-1}$

since $E|\varepsilon^2| = E(\varepsilon^2) = E(E(\varepsilon^2|x)) \overset{E(\varepsilon|x)=0}{=} E(Var(\varepsilon|x)) = \sigma^2 < \infty$

A1 $E|x'\varepsilon| < \infty \qquad$ and then $E(x'\varepsilon) = E(E(x'\varepsilon|x)) \overset{E(\varepsilon|x)=0}{=} 0$

A2 $E|x'x| < \infty \qquad$ then $E(x'x) = Q > 0$ and $E(x'x)^{-1} = Q^{-1} < \infty$

$\sqrt{n}(\hat{\sigma}^2 - \sigma^2) = \sqrt{n}\frac{n}{n-k}\left(\frac{\Sigma\varepsilon'\varepsilon}{n} - \sigma^2\right) + \frac{k\sqrt{n}}{n-k}\sigma^2 - \frac{n}{n-k}\sqrt{n}\left(\frac{\Sigma x'\varepsilon}{n}\right)'\left(\frac{\Sigma x'x}{n}\right)^{-1}\left(\frac{\Sigma x'\varepsilon}{n}\right)$

$\frac{n}{n-k} \xrightarrow{p} 1 \qquad \frac{k\sqrt{n}}{n-k}\sigma^2 \xrightarrow{p} 0 \qquad$ by c.l.t. $\sqrt{n}\left(\frac{\Sigma\varepsilon'\varepsilon}{n} - \sigma^2\right) \xrightarrow{d} N\left(0, E(\varepsilon^4) - \sigma^4\right)$

A3 $E\left|\varepsilon^4\right| < \infty$ by c.l.t. $\sqrt{n}\frac{\Sigma x'\varepsilon}{n} \xrightarrow{d} N\left(0, E\left(x'x\right)\sigma^2\right)$ by l.l.n.+contin $\left(\frac{\Sigma x'x}{n}\right)^{-1} \xrightarrow{p} E\left(x'x\right)^{-1}$

Hence, by Slutsky $\sqrt{n}\left(\hat{\sigma}^2 - \sigma^2\right) \xrightarrow{d} N\left(0, E\left(\varepsilon^4\right) - \sigma^4\right)$ is normal.

Need a consistent estimator for $E\left(\varepsilon^4\right) - \sigma^4$.

$\frac{\Sigma}{n}e^4 = \frac{\Sigma}{n}\left[\varepsilon^4 + 4\varepsilon^3 x(\beta - \hat{\beta}) + 6\varepsilon^2\left(x(\beta - \hat{\beta})\right)^2 + 4\varepsilon\left(x(\beta - \hat{\beta})\right)^3 + \left(x(\beta - \hat{\beta})\right)^4\right] =$

$\frac{\Sigma}{n}\varepsilon^4 + 4\frac{\Sigma}{n}\varepsilon^3 x(\beta - \hat{\beta}) + 6\frac{\Sigma}{n}\varepsilon^2\left[x(\beta - \hat{\beta})\right]^2 + 4\frac{\Sigma}{n}\varepsilon\left[x(\beta - \hat{\beta})\right]^3 + \frac{\Sigma}{n}\left[x(\beta - \hat{\beta})\right]^4$

We know that $\hat{\beta}$ is consistent, i.e. $\beta - \hat{\beta} = o_p(1)$. Assuming also:

A4 $E\left|x\varepsilon^3\right| < \infty$ **A5** $E\left|x^2\varepsilon^2\right| < \infty$ **A6** $E\left|x^3\varepsilon\right| < \infty$ **A7** $E\left|x^4\right| < \infty$

It is then straightforward that by l.l.n. the averages of moments converge to their expectations, and by continuity, consistency of OLS and Slutsky $\frac{\Sigma}{n}e^4 \xrightarrow{p} E\varepsilon^4$.

Therefore, $\left[\frac{1}{n}\Sigma e^4 - \left(\frac{\Sigma e^2}{n-k}\right)^2\right] \xrightarrow{p} E\left(\varepsilon^4\right) - \sigma^4$ is a consistent estimator of the variance.

Exercise 4

$\hat{\beta}_{OLS} = \beta + \left(\frac{1}{n}\Sigma X_i'X_i\right)\left(\frac{1}{n}\Sigma X_i'\varepsilon_i\right)$

A1 $\lim\left(\frac{1}{n}\Sigma X_i'X_i\right) = Q > 0 \quad \Rightarrow$ by continuity $\exists \lim\left(\frac{1}{n}\Sigma X_i'X_i\right)^{-1} = Q^{-1}$

$E\left(X_i'\varepsilon_i\right) = X_i'E\left(\varepsilon_i\right) = 0 \qquad Var\left(X_i'\varepsilon_i\right) = X_i'Var\left(\varepsilon_i\right)X_i = \sigma^2 Q$

Since X_i is a constant and ε_i are i.i.d. $X_i'\varepsilon_i$ are independent. Since variance is finite $\exists\delta = 1$,s.t. $\Sigma E\left(\left|X_i'\varepsilon_i/i\right|^{1+\delta}\right) = \frac{X_i'X_i\sigma^2}{6/\pi^2} < \infty$ and Markov theorem applies:

$\lim\left(\frac{1}{n}\Sigma X_i'\varepsilon_i\right) = E\left(X_i'\varepsilon_i\right) = 0$. Hence, by Slutsky $\hat{\beta}_{OLS} \xrightarrow{p} \beta + Q^{-1} * 0 = \beta$. $\hat{\beta}_{OLS}$ is consistent.

$\sqrt{n}\left(\hat{\beta}_{OLS} - \beta\right) = \left(\frac{1}{n}\Sigma X_i'X_i\right)\left(\frac{1}{\sqrt{n}}\Sigma X_i'\varepsilon_i\right)$

A2 $\exists\delta > 0$,s.t. $\Sigma E\left(\left|X_i'\varepsilon_i\right|^{2+\delta}\right) < \Delta < \infty$. Then Liapunov theorem applies:

$\frac{1}{\sqrt{n}}\Sigma X_i'\varepsilon_i \xrightarrow{d} N\left(0, \sigma^2 Q\right)$. Hence, by Slutsky $\sqrt{n}\left(\hat{\beta}_{OLS} - \beta\right) \xrightarrow{d} N\left(0, \sigma^2 Q^{-1}\right)$.

Exercise 5

(a) GLS: Transform the equation dividing all observations by σ_g. Then the new variables satisfy CLR and can be estimated by OLS: $\hat{\beta}_{GLS} = \left(\Sigma_g\frac{X_g'X_g}{\sigma_g^2}\right)^{-1}\left(\Sigma_g\frac{X_g'Y_g}{\sigma_g^2}\right)$.

FGLS: Run n_g separate groupwise regressions. Take $\hat{\sigma}_g^2 = \frac{\hat{e}_g'\hat{e}_g}{n_g - k}$ and plug into formula above:

$\hat{\beta}_{FGLS} = \left(\Sigma_g\frac{X_g'X_g}{\hat{\sigma}_g^2}\right)^{-1}\left(\Sigma_g\frac{X_g'Y_g}{\hat{\sigma}_g^2}\right)$. Repeat untill convergence.

ML: $\ln L = -\frac{n}{2}\ln 2\pi - \frac{1}{2}\Sigma_g n_g \ln\sigma_g^2 - \Sigma_g\frac{\Sigma_{i=1}^{n_g}(y_{i,g} - x_{i,g}\beta)^2}{2\sigma_g^2}$

$\frac{\partial L}{\partial\beta} = \Sigma_g\frac{\Sigma_{i=1}^{n_g}\left(x_{i,g}'y_{i,g} - x_{i,g}'x_{i,g}\beta\right)}{\sigma_g^2} = 0 \qquad \hat{\beta}_{MLE} = \left(\Sigma_g\frac{X_g'X_g}{\hat{\sigma}_g^2}\right)^{-1}\left(\Sigma_g\frac{X_g'Y_g}{\hat{\sigma}_g^2}\right)$

$\frac{\partial L}{\partial\sigma_g^2} = -\Sigma_g\frac{n_g}{2\sigma_g^2} - \Sigma_g\frac{\Sigma_{i=1}^{n_g}\hat{e}_g'\hat{e}_g}{2\left(\sigma_g^2\right)^2} = 0 \qquad \hat{\sigma}_{gMLE}^2 = \frac{\hat{e}_g'\hat{e}_g}{n_g}$ (the only difference from FGLS is k).

(b) LR: UR: $\ln L = -\frac{n}{2}\ln 2\pi - \frac{1}{2}\Sigma_g n_g\ln\frac{\hat{e}_g'\hat{e}_g}{n_g} - \frac{n}{2}$

R: $\ln L = -\frac{n}{2}\ln 2\pi - \frac{1}{2}n\ln\frac{\hat{e}'\hat{e}}{n} - \frac{n}{2}$ which is based on standard $\hat{\beta}_{OLS} = \left(X'X\right)^{-1}\left(X'Y\right)$.

$LR = -2(\ln L_R - \ln L_{UR}) = \left[n\ln\frac{\hat{e}'\hat{e}}{n} - \Sigma_g n_g\ln\frac{\hat{e}_g'\hat{e}_g}{n_g}\right] \sim \chi^2(G)$

a

	n	sigma	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	100	0.5	0.051	0.102	0.0031	0.0038	0.0521	0.0341
White	100	0.5	0.054	0.109	0.0031	0.0038	0.0521	0.0341
GLS	100	0.5	0.051	0.102	0.0031	0.0038	0.0521	0.0341
FGLS	100	0.5	0.071	0.131	0.0033	0.0035	0.0544	0.0364

	n	sigma	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	400	0.5	0.048	0.104	-0.0007	-0.0006	0.0252	0.0167
White	400	0.5	0.051	0.113	-0.0007	-0.0006	0.0252	0.0167
GLS	400	0.5	0.048	0.104	-0.0007	-0.0006	0.0252	0.0167
FGLS	400	0.5	0.053	0.109	-0.0007	-0.0003	0.0255	0.017

	n	sigma	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	1600	0.5	0.054	0.106	-0.0001	0.0003	0.0122	0.008
White	1600	0.5	0.055	0.104	-0.0001	0.0003	0.0122	0.008
GLS	1600	0.5	0.054	0.106	-0.0001	0.0003	0.0122	0.008
FGLS	1600	0.5	0.051	0.105	-0.0001	0.0004	0.0122	0.0081

	n	sigma	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	100	1	0.069	0.109	0.003	0.0017	0.1006	0.0669
White	100	1	0.066	0.117	0.003	0.0017	0.1006	0.0669
GLS	100	1	0.069	0.109	0.003	0.0017	0.1006	0.0669
FGLS	100	1	0.076	0.126	0.0018	0.0015	0.1068	0.0703

	n	sigma	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	400	1	0.053	0.095	-0.0011	-0.0029	0.0504	0.0337
White	400	1	0.054	0.097	-0.0011	-0.0029	0.0504	0.0337
GLS	400	1	0.053	0.095	-0.0011	-0.0029	0.0504	0.0337
FGLS	400	1	0.054	0.104	-0.0013	-0.0023	0.0505	0.0341

	n	sigma	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	1600	1	0.061	0.108	0.0009	-0.0004	0.0249	0.016
White	1600	1	0.056	0.109	0.0009	-0.0004	0.0249	0.016
GLS	1600	1	0.061	0.108	0.0009	-0.0004	0.0249	0.016
FGLS	1600	1	0.055	0.113	0.0009	-0.0002	0.025	0.0162

	n	sigma	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	100	2	0.046	0.096	0.0047	0.0136	0.2072	0.1413
White	100	2	0.05	0.101	0.0047	0.0136	0.2072	0.1413
GLS	100	2	0.046	0.096	0.0047	0.0136	0.2072	0.1413
FGLS	100	2	0.057	0.123	0.0055	0.0075	0.2121	0.145

	n	sigma	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	400	2	0.032	0.087	-0.0047	-0.0067	0.0979	0.0698
White	400	2	0.032	0.093	-0.0047	-0.0067	0.0979	0.0698
GLS	400	2	0.032	0.087	-0.0047	-0.0067	0.0979	0.0698
FGLS	400	2	0.032	0.093	-0.0049	-0.0043	0.0984	0.071

	n	sigma	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	1600	2	0.055	0.099	-0.0004	0.0004	0.0493	0.0345
White	1600	2	0.055	0.103	-0.0004	0.0004	0.0493	0.0345
GLS	1600	2	0.055	0.099	-0.0004	0.0004	0.0493	0.0345
FGLS	1600	2	0.045	0.099	-0.0005	-0.0001	0.0493	0.0343

b

	n	c	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	100	1	0.21	0.286	0.0006	0.0035	0.1703	0.1085
White	100	1	0.064	0.116	0.0006	0.0035	0.1703	0.1085
GLS	100	1	0.06	0.1	0.0029	0.0039	0.1014	0.0697
FGLS	100	1	0.443	0.488	-0.0002	0.0044	0.3218	0.1586

	n	c	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	400	1	0.202	0.29	0.0033	0	0.0841	0.0575
White	400	1	0.049	0.097	0.0033	0	0.0841	0.0575
GLS	400	1	0.054	0.104	0.0008	0.001	0.05	0.0325
FGLS	400	1	0.417	0.482	0.0004	-0.0004	0.1962	0.0816

	n	c	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	1600	1	0.191	0.268	-0.0008	-0.0015	0.0418	0.0266
White	1600	1	0.045	0.1	-0.0008	-0.0015	0.0418	0.0266
GLS	1600	1	0.047	0.1	-0.0001	-0.0009	0.0247	0.0166
FGLS	1600	1	0.272	0.344	0.002	-0.0006	0.08	0.0302

	n	c	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	100	2	0.22	0.289	-0.0031	-0.0061	0.1219	0.0785
White	100	2	0.066	0.126	-0.0031	-0.0061	0.1219	0.0785
GLS	100	2	0.053	0.106	-0.0028	-0.0013	0.0743	0.0489
FGLS	100	2	0.419	0.488	-0.0006	-0.0038	0.2105	0.1101

	n	c	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	400	2	0.193	0.273	-0.0021	-0.0015	0.0597	0.0417
White	400	2	0.049	0.088	-0.0021	-0.0015	0.0597	0.0417
GLS	400	2	0.053	0.105	-0.0014	-0.0012	0.0349	0.0235
FGLS	400	2	0.366	0.421	-0.0024	-0.0028	0.1325	0.0527

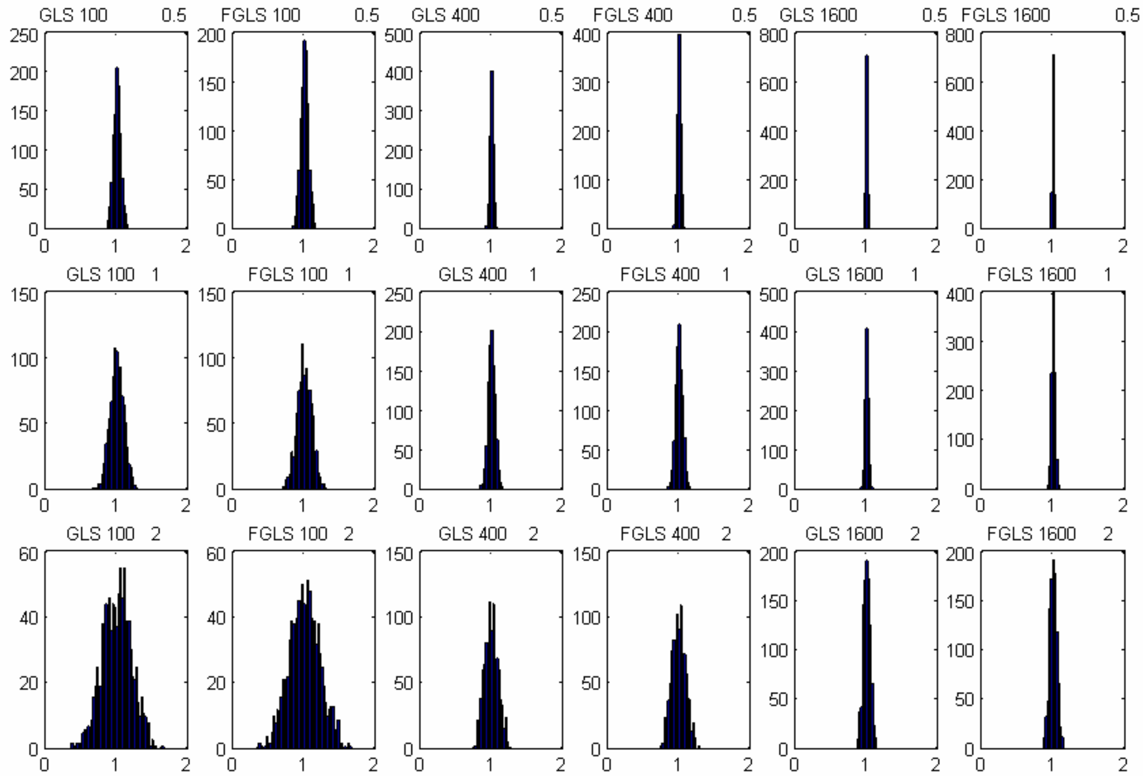
	n	c	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	1600	2	0.212	0.284	0.0012	0.002	0.0297	0.0206
White	1600	2	0.054	0.106	0.0012	0.002	0.0297	0.0206
GLS	1600	2	0.038	0.088	0	-0.0005	0.0174	0.0116
FGLS	1600	2	0.283	0.362	0.0019	0.0017	0.064	0.0229

	n	c	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	100	4	0.201	0.281	-0.0025	-0.0027	0.083	0.0539
White	100	4	0.053	0.105	-0.0025	-0.0027	0.083	0.0539
GLS	100	4	0.051	0.089	-0.0008	-0.0009	0.0478	0.0316
FGLS	100	4	0.424	0.49	-0.005	-0.0034	0.1506	0.0783

	n	c	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	400	4	0.208	0.3	-0.0007	0.0003	0.0437	0.0304
White	400	4	0.061	0.107	-0.0007	0.0003	0.0437	0.0304
GLS	400	4	0.049	0.107	0.0001	-0.0011	0.0248	0.0175
FGLS	400	4	0.4	0.484	-0.0001	0.0009	0.1007	0.0409

	n	c	Reject 5%	Reject 10%	Mean bias	Med bias	RMSE	MedAE
OLS	1600	4	0.219	0.289	-0.0004	-0.0004	0.0217	0.0146
White	1600	4	0.063	0.116	-0.0004	-0.0004	0.0217	0.0146
GLS	1600	4	0.047	0.105	-0.0002	-0.0002	0.0124	0.0087
FGLS	1600	4	0.286	0.35	-0.0014	-0.0009	0.0333	0.0162

Distributions of estimates GLS and FGLS for Model 1.



Distributions of estimates GLS and FGLS for Model 2.