**Exercise 7** *If* $(1-2t)^{-6}$ *is the moment generating function of the random variable* $X$*, find* $\Pr(X < 21.026)$

***Proof.*** $\beta = 2, \qquad \alpha = 6, \qquad \Pr(X < 21.026) = \int_0^{21.026} \frac{1}{5!2^6} x^{6-1} e^{-x/2} dx = 0.95$ ■

**Exercise 8** *If* $X$ *has a gamma distwith* $\alpha = 3$ *and* $\beta = 4$*, find* $\Pr(4.408 < X < 25.184)$*.*

***Proof.*** $\Pr(4.408 < X < 25.184) = \int_{4.408}^{25.184} \frac{1}{2!4^3} x^{3-1} e^{-x/4} dx = 0.85$ ■

**Exercise 9** *Compute the skewness and kurtosis of a gamma distribution with parameters* $\alpha$ *and* $\beta$*.*

***Proof.*** $E[X^2] = \frac{\partial^2}{\partial t^2}(\frac{1}{(1-\beta t)^\alpha})\Big|_{t=0} = \alpha\beta^2 + \alpha^2\beta^2$

$E[X^3] = \frac{\partial^3}{\partial t^3}(\frac{1}{(1-\beta t)^\alpha})\Big|_{t=0} = 2\alpha\beta^3 + 3\alpha^2\beta^3 + \alpha^3\beta^3$

$E[X^4] = \frac{\partial^4}{\partial t^4}(\frac{1}{(1-\beta t)^\alpha})\Big|_{t=0} = 6\alpha\beta^4 + 11\alpha^2\beta^4 + 6\alpha^3\beta^4 + \alpha^4\beta^4$

$E\left[(X-\mu)^3\right]/\sigma^3 = \frac{1}{\alpha^{3/2}\beta^3}(2\alpha\beta^3 + 3\alpha^2\beta^3 + \alpha^3\beta^3 - 3\alpha\beta(\alpha\beta^2 + \alpha^2\beta^2) + 3\alpha^3\beta^3 - \alpha^3\beta^3) = \frac{2}{\sqrt{\alpha}}$

$E\left[(X-\mu)^4\right]/\sigma^4 = \frac{1}{\alpha^2\beta^4}(6\alpha\beta^4 + 11\alpha^2\beta^4 + 6\alpha^3\beta^4 + \alpha^4\beta^4 - 4\alpha\beta * (2\alpha\beta^3 + 3\alpha^2\beta^3 + \alpha^3\beta^3) + 6\alpha^2\beta^2 * (\alpha\beta^2 + \alpha^2\beta^2) - 4\alpha^4\beta^4 + \alpha^4\beta^4) = 3\frac{2+\alpha}{\alpha}$ ■

**Exercise 10** *Let* $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$ *denote the CDF of* $N(0,1)$*. Show that* $\Phi(-z) = 1 - \Phi(z)$

***Proof.*** $\Phi(-z) = \int_{-\infty}^{-z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = 1 - \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = 1 - \Phi(z)$ ■

**Exercise 11** *Suppose that* $X \sim N(\mu, \sigma^2)$*. Find* $b$ *such that* $\Pr[-b < (X-\mu)/\sigma < b] = 0.90$

***Proof.*** $X \sim N(\mu, \sigma^2) \Rightarrow Y = \frac{(X-\mu)}{\sigma} \sim N(0,1) \Rightarrow$

$\Pr[|Y| < b] = 0.90 \qquad \Rightarrow \qquad b = 1.64485$ ■

**Exercise 12** *Compute the skewness and kurtosis of* $N(\mu, \sigma^2)$

***Proof.*** $E[X^2] = \frac{\partial^2}{\partial t^2}(\exp\left[\mu t + \frac{\sigma^2}{2}t^2\right])\Big|_{t=0} = \sigma^2 + \mu^2$

$E[X^3] = \frac{\partial^3}{\partial t^3}(\exp\left[\mu t + \frac{\sigma^2}{2}t^2\right])\Big|_{t=0} = \mu^3 + 3\sigma^2\mu$

$E[X^4] = \frac{\partial^4}{\partial t^4}(\exp\left[\mu t + \frac{\sigma^2}{2}t^2\right])\Big|_{t=0} = 3\sigma^4 + \mu^4 + 6\sigma^2\mu^2$

$E\left[(X-\mu)^3\right]/\sigma^3 = \frac{1}{\sigma^3}(\mu^3 + 3\sigma^2\mu - 3\mu\sigma^2 - 3\mu^3 + 3\mu^3 - \mu^3) = 0$

$E\left[(X-\mu)^4\right]/\sigma^4 = \frac{1}{\sigma^4}(3\sigma^4 + \mu^4 + 6\sigma^2\mu^2 - 4\mu^4 - 12\sigma^2\mu^2 + 6\mu^2\sigma^2 + 6\mu^4 - 4\mu^4 + \mu^4) = 3$ ■