Homework 1

Econ 202C

Due May 23, 2006, 5pm

There are three problems. You should attempt all of them, even if you do not manage to solve them completely. You are encouraged to work in groups, but should write down your answers independently.

Problem 1: A shopping time model of money

Consider the following model of a monetary economy: There is a representative household, who in each period chooses to consumption c_t, labour supply l_t and real balances m_t. Purchasing consumption takes time and real balances; it is assumed that if the household chooses to consume c and hold real balances m, the time spent shopping is $g(c,m)$, with $g_c > 0$; $g_m < 0$; $g_{cc} > 0$; $g_{mm} > 0$ and $g_{cm} > 0$. In addition, $\lim_{m\to\infty} g(c,m) = 0$. The household thus maximizes $\sum_{t=0}^{\infty} \beta^t u\left(c_t, \widehat{l}_t\right)$, s.t. $\widehat{l}_t = l_t + g(c_t, m_t)$, subject to his budget constraint in each period. We also assume $u_c > 0$; $u_{cc} < 0$; $u_l < 0$; $u_{ll} > 0$ and $u_{cl} = 0$. and assume that $\beta < 1$. Output is produced from capital and labor, according to a production function with constant returns to scale. Capital fully depreciates in each period. Finally, there is a monetary authority that in each period makes a lumpsum transfer or tax T_t to the household. There is perfect foresight of all future transfers.

(a) Write down the representative household's intertemporal optimization problem.

(b) Define a competitive equilibrium for this economy.

(c) Derive the First-order conditions for consumption, capital, real money, balances and labor supply.

(d) Now, assume that the government follows a policy of constant money growth, and let μ denote the money growth rate, i.e. $T_t = \mu M_{t-1}$. Derive the conditions characterizing steady-state equilibrium allocations.

(e) What are the real effects of money growth? Is there a dichotomy between real and nominal variables? What is the optimal money growth rate in this economy?

Problem 2: Money in the utility function

Consider the model of money in the utility function that we have seen in class. Suppose that the money supply is fixed. Discuss whether in addition to the stationary equilibrium that we have seen in class, this model also admits non-stationary equilibria. In particular, can there be hyper-inflationary equilibria, or equilibria with deflations?

You should ideally attempt to provide a formal analysis, imposing additional conditions on primitives (preferences, technology), if necessary. Alternatively, you may also discuss this problem by checking numerically whether such non-stationary equilibria exist.

Problem 3: A model with two moneys

Consider the endowment economy we have considered in class in the first two lectures. There are two types of consumers, odd and even, with the even (odd) types being endowed with one unit of consumption in even (odd) periods, and zero units of consumption in odd (even) periods. There is a measure 1 of each type. Preferences are as in the lectures.

Markets: There are two fiat currencies in fixed supply M and N. Let q_t^M and q_t^N denote the prices, i.e. the real values of these currencies in terms of consumption at date t. At each date t, the consumers can trade with each other in consumption goods and the two currencies.

(a) Define the households' optimization problem, and the competitive equilibrium with two moneys.

(b) Characterize steady-state equilibria of this economy. Is it possible that both moneys have positive value? What can you say about the relative value of one currency vs. the other (or the exchange rate between the currencies) (i) over time? (ii) across equilibria?

(c) Let $s_t \in \{L, H\}$ be an iid random variable, with $\Pr(H) = \Pr(L) = 1/2$ in each period. s_t is a 'sunspot' variable, which does not affect any of the model primitives, which are fully deterministic. Can you construct sunspot equilibria, in which the exchange rate between the currencies the two currencies is stochastic and depends on the history s^t of current or past sunspot realizations? If you cannot construct such equilibria, can you rule them out, i.e. show that in all equilibria, prices must be deterministic?