

# Problem Set 1

## The optimal growth model with preference shocks

A representative household maximizes:

$$\max E \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\alpha_t (c_t^{1-\sigma} - 1)}{1-\sigma} \right\}$$

The size of the population is  $N_t$ . The technology and the equations for technological change and population are:

$$AK_t^\theta (X_t N_t)^{1-\theta} + (1-\delta)K_t \geq C_t + K_{t+1}, 0 < \theta \leq 1$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

$$X_t = \gamma X_{t-1}, X_0 = 1, \gamma > 1$$

$$N_t = \kappa N_{t-1}, N_0 = 1, \kappa > 1$$

The preference shock takes on 3 values:

$$\alpha \in \{.75, 1.00, 1.25\}$$

The Markov transition matrix is given by:

$$\begin{vmatrix} .15 & .7 & .15 \\ .1 & .8 & .1 \\ .15 & .7 & .15 \end{vmatrix}$$

(1) Show that as  $\sigma \rightarrow 1$ ,  $\frac{(c_t^{1-\sigma}-1)}{1-\sigma} \rightarrow \ln(c_t)$ . From here on, assume logarithmic preferences over consumption. Show how to detrend this model so that it possesses a steady state in per-capita, transformed variables (here, assume the non-stochastic version of the economy in which  $\alpha = 1$ ).

(2) Choose parameter values such that the steady state per-capita capital stock = 1, the per-period growth rate of  $X$  is 2%, the per-period growth rate of the population is 1.5%,  $2/3$  of output is paid to labor, the steady state real return to capital (net of depreciation) is 8%, and the investment/output ratio = 25%.

(3) Define an RCE for this economy.

(4) Given the three values for  $\alpha$ , define a grid of 101 points on the capital stock, 50 points below the steady state value of 1 and 50 points above the steady state value of 1. Let the distance between the points be 0.005. Using the planner's problem, iterate on this discretized model until the value function converges, and report the solution. Calculate the correlation between consumption and investment. The easiest way to do this is to simulate the economy over many periods, say 1,000, and calculate the statistic for the simulate observations, discarding the first 100 observations.

(5) Repeat (4), but assume that all of the elements of the transition matrix on the principal diagonal are .8, and all other elements are .1. Re-calculate the correlation between consumption and investment. How does this correlation depend on the transition probabilities, and why does the correlation change?

### **Asset Pricing and the Equity Premium**

Replicate Mehra and Prescott's 1985 analysis, using data from 1950 through 2005. Specifically, calculate the real return to equity and to a government bond for the risk aversion values used by Mehra and Prescott. Describe and report the source of the data that you use as well as these returns and the equity premium. (Hint: Robert Shiller may have data that can be useful for this exercise).