Econ202C Homework #3 Anton Cheremukhin

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**Exercise 1** *Shopping time model of money*

HH:$\max_{c,n,m} \Sigma_{t=0}^{\infty}\beta^t u\left(c_t, 1-n_t-g\left(c_t, \frac{M_t}{p_t}\right)\right)$ s.t. $p_t c_t + p_t k_{t+1} + M_t \le R_t k_t + W_t n_t + T_t + M_{t-1}$

Define $\quad X_t/p_t = x_t \quad \frac{p_{t-1}}{p_t} = \frac{1}{1+\pi_t}$

a) **HH**: $\max_{c,n,m} \Sigma_{t=0}^{\infty}\beta^t u\left(c_t, 1-n_t-g\left(c_t, m_t\right)\right)$ s.t. $c_t + k_{t+1} + m_t \le r_t k_t + w_t n_t + \tau_t + \frac{m_{t-1}}{1+\pi_t}$

**F**: $\max_{k,l} y_t - r_t\widetilde{k}_t - w_t\widetilde{n}_t$ s.t. $y_t = f\left(\widetilde{k}_t, \widetilde{n}_t\right)$ - CRS

Market clearing: $\widetilde{n}_t = n_t, \quad c_t + k_{t+1} = y_t, \quad k_t = \widetilde{k}_t \quad m_t = \tau_t + \frac{m_{t-1}}{1+\pi_t}$

b) **CE**: allocation $\left\{c_t, m_t, n_t, k_{t+1}, \widetilde{k}_t, \widetilde{n}_t, y_t\right\}$ and prices $\{1, \pi_t, r_t, w_t\}$ s.t.

1) alloc solves HH given prices 2)alloc solves F given prices 3)markets clear

c) $L = \Sigma_{t=0}^{\infty}\beta^t\left[u\left(c_t, 1-n_t-g\left(c_t, m_t\right)\right) + \lambda_t\left(r_t k_t + w_t n_t + \tau_t + \frac{m_{t-1}}{1+\pi_t} - c_t - k_{t+1} - m_t\right)\right]$

FOC$_{c_t}$: $u'_c(t) - u'_l(t)\, g'_c(t) = \lambda_t$ FOC$_{n_t}$: $u'_l(t) = \lambda_t w_t$

FOC$_{m_t}$: $u'_l(t)\, g'_m(t) + \lambda_t = \frac{\beta}{1+\pi_t}\lambda_{t+1}$ FOC$_{k_{t+1}}$: $\lambda_t = \beta\lambda_{t+1} r_{t+1}$

FOC$_{\widetilde{k}_t}$: $f'_k\left(\widetilde{k}_t, \widetilde{n}_t\right) = r_t$ FOC$_{\widetilde{n}_t}$: $f'_n\left(\widetilde{k}_t, \widetilde{n}_t\right) = w_t$

d) Monetary policy: $T_t = \mu M_{t-1} \quad M_t = T_t + M_{t-1}$

Hence, $\tau_t = \mu\frac{m_{t-1}}{1+\pi_t} \quad m_t = \tau_t + \frac{m_{t-1}}{1+\pi_t} = \frac{(1+\mu)}{1+\pi_t}m_{t-1}$

**Steady-state**: all small variables constant: $\mu = \pi_t$

$u'_l = \left(u'_c - u'_l g'_c\right) f'_n \qquad -u'_l g'_m = \left(1 - \frac{\beta}{1+\mu}\right)\left(u'_c - u'_l g'_c\right) \qquad 1 = \beta f'_k \qquad k + c = f$

$u\left(c, 1-n-g\left(c,m\right)\right) \qquad g\left(c,m\right) \qquad f\left(k,n\right) \qquad \Rightarrow \qquad$ Have four equations in $\{c, k, n, m\}$.

e) The money has real effects. A growth in real money balances reduces time spent on shopping, therefore increasing both leisure and labor supply, which increases consumption. So an increase in real money balances has a positive effect on wealth. There is no dichotomy.

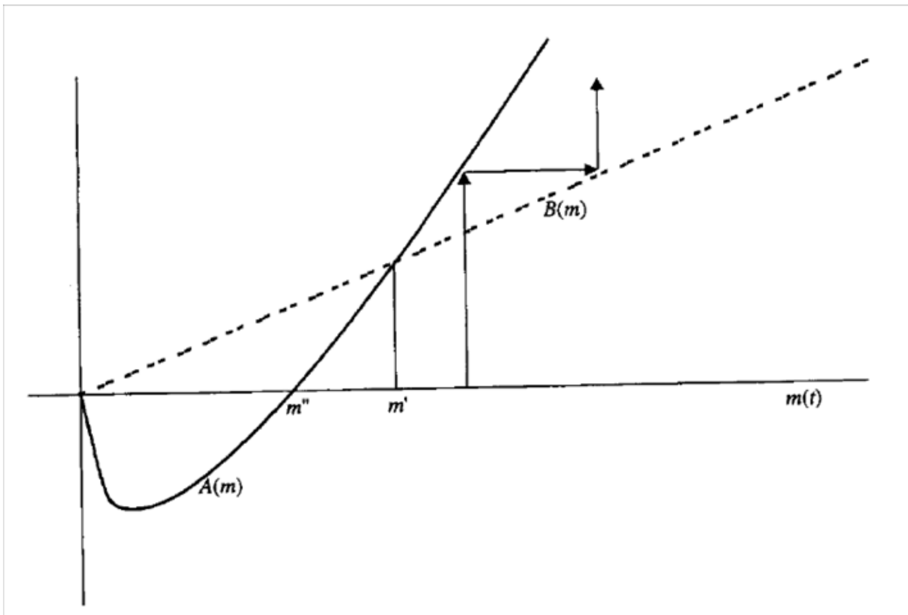
We can regard $\left(1 - \frac{\beta}{1+\mu}\right)$ as $\left(1 - \frac{1}{(1+\mu)r}\right) = \left(1 - \frac{1}{1+i}\right) = \frac{i}{1+i}$, where $i$-nominal interest rate. Then we can rewrite the second equation as $-wg'_m = \frac{i}{1+i}$. The l.h.s. represents the marginal value of time, saved by holding additional real money balances. Social optimality requires that we maximize this time, which is the only channel through which money affects the economy. The more money we hold, the more time we save, the closer we are to social optimality. Hence, social optimality requires that nominal interest rate be equal to zero: $(1+\mu) = \frac{1}{r} = \beta$ - the Friedman rule.

**Exercise 2** *Money in the utility function.*

We assume for simplicity (following Brock (1974), Obstfeld and Rogoff (1983) and (1986), and Walsh (2003)) that the utility function is separable: $u(c,m) = v(c) + \phi(m)$. Then, dynamics of real balances around the steady-state in the model are described by:

$B\left(m_{t+1}\right) \equiv \frac{\beta}{1+\mu}v_c\left(c_{ss}\right)m_{t+1} = \left[v_c\left(c_{ss}\right) - \phi_m\left(m_t\right)\right]m_t \equiv A\left(m_t\right)$

If $\lim\limits_{m\to 0}\phi_m m = 0$, then the functions look like this:




There are two steady-states, 0 and $m'$. Values starting to the right of $m'$ can be ruled out, because this would imply that real money balances go to infinity, which violates the transversality condition. Some values starting to the left of $m'$ imply paths converging to 0. This is a situation of hyperinflation. So, we either have a steady-state or a hyperinflation. No deflations are allowed.

**Exercise 3** *A model with two moneys*

HH: $\max \Sigma_{t=0}^{\infty} \beta^t u\left(c_t^i\right)$ s.t. $c_t^i + q_t^M m_t^i + q_t^N n_t^i \leq y_t^i + q_t^M m_{t-1}^i + q_t^N n_{t-1}^i \qquad m_t \geq 0 \qquad n_t \geq 0$
Market clearing: $c_t^o + c_t^e = 1, \qquad m_t^o + m_t^e = M, \qquad n_t^o + n_t^e = N.$
Endowments: $y_t^e = \{1, 0, 1, ...\} \qquad y_t^o = \{0, 1, 0, ...\} \qquad m_{-1}^o = M, \qquad n_{-1}^o = N.$
**CE**: allocations $\left\{c_t^i, m_t^i, n_t^i\right\}$ and prices $\left\{q_t^M, q_t^N\right\}$,
s.t. allocations solve HH given prices and endowments, and markets clear.
$\max \Sigma_{t=0}^{\infty} \beta^t \left[u\left(c_t^i\right) + \lambda_t \left(y_t^i + \left\{q_t^M m_t^i + q_t^N n_t^i\right\} - c_t^i - \left\{q_t^M m_{t-1}^i + q_t^N n_{t-1}^i\right\}\right) + \mu_t m_t^i + \nu_t n_t^i\right]$
FOC$_{c_t}$:$u'\left(c_t^i\right) = \lambda_t \qquad$ FOC$_{m_t}$: $q_t^M \lambda_t + \mu_t = \beta q_{t+1}^M \lambda_{t+1} \qquad$ FOC$_{n_t}$: $q_t^N \lambda_t + \mu_t = \beta q_{t+1}^N \lambda_{t+1}$
$q_t^M u'\left(c_t^i\right) + \mu_t = \beta q_{t+1}^M u'\left(c_{t+1}^i\right) \qquad q_t^N u'\left(c_t^i\right) + \mu_t = \beta q_{t+1}^N u'\left(c_{t+1}^i\right)$
In each period for one of the guys both multipliers will be zero $\Rightarrow \quad \frac{q_t^M}{q_{t+1}^M} = \frac{q_t^N}{q_{t+1}^N}$.
I.e. the exchange rate is constant over time in general: $\frac{q_t^M}{q_t^N} = \frac{q_{t+1}^M}{q_{t+1}^N} = e$
Can redefine $q_t^M m_t^i + q_t^N n_t^i = q_t^N \left(eM + N\right) = \frac{1}{p_t}\widetilde{M}$. This leads us to the initial model.
The value of the exchange rate in equilibrium is undetermined: $q_t^N \left(eM + N\right) = 1 - \overline{c}_t$

We have one equation in two unknowns. Therefore the exchange rate can take any initial value, but cannot change over time.

Let's introduce a sunspot $s_t$ - an irrelevant random variable, and let's allow everything in our economy depend on the state $s_t$. Then solving the corresponding expected utility maximization problem we get the Euler equations of the form:

$E_t\left(q_t^j\left(s_t\right) u'\left(c_t^i\left(s_t\right)\right) + \mu_t\left(s_t\right) - \beta q_{t+1}^j\left(s_{t+1}\right) u'\left(c_{t+1}^i\left(s_{t+1}\right)\right) | \Omega_t\right) = 0$ for any i and j.

We can easily see that in each period for one of the guys multiplies for both moneys should be zero: $E_t\left(q_t^j\left(s_t\right) u'\left(c_t^i\left(s_t\right)\right) - \beta q_{t+1}^j\left(s_{t+1}\right) u'\left(c_{t+1}^i\left(s_{t+1}\right)\right) | \Omega_t\right) = 0$ for some i for any j. I.e. sunspots "don't matter", since they don't affect consumption decisions.

If $s_t \in \Omega_t$ then today's **realized** price has to be equal to tomorrow's **expected** price. In this case the exchange rate again has to be constant over time, and sunspots do not affect any of the variables.

The case $s_t \notin \Omega_t$ mean agents first make their decisions conditional on the realization of the sunspot, than the sunspot is realized, and afterwards the decisions are implemented. In this case today's **expected** price has to be equal to tomorrow's **expected** price. In the case of an i.i.d. sunspot this is trivially satisfied. Hence, if today's sunspot is unknown at the moment agents make their decisions, the exchange rate can be any function of the sunspot in equilibrium.

We interpret this timing as if agents were trading contingent claims on money dependent on the realization of the sunspot. This mechanism actually requires commitment of the agents to those claims. So, for sunspots to affect the exchange rate, we need perfect enforcement on these state-contingent claims, though in the initial model we assumed that money can only have positive value, because of lack of commitment.