Econ202C Homework #2 Anton Cheremukhin

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Exercise 1

Sequential problem:
$\max \Sigma_{t=0}^{\infty} \beta^t \left[\ln C_t + \phi \ln (1 - l_{ct} - l_{kt})\right] \qquad X_{ct} = \exp(\gamma_c t) \qquad X_{kt} = \exp(\gamma_k t)$
$K_{ct}^{\theta} (X_{ct} l_{ct})^{1-\theta} \geq C_t \qquad K_{kt}^{\theta} (X_{kt} l_{kt})^{1-\theta} \geq I_{ct} + I_{kt}$
$I_{ct} = K_{ct+1} - (1-\delta) K_{ct} \geq 0 \qquad I_{kt} = K_{kt+1} - (1-\delta) K_{kt} \geq 0$
Growth rates: $\theta d \ln K_c + (1-\theta) d \ln X_c = d \ln C, \qquad \theta d \ln K_k + (1-\theta) d \ln X_k = d \ln I$
$\frac{I_c}{K_c} = \frac{K_{ct+1}}{K_{ct}} - (1-\delta) = const \qquad \frac{I_k}{K_k} = \frac{K_{kt+1}}{K_{kt}} - (1-\delta) = const$
Hence, $d \ln I_c = d \ln K_c = d \ln I_k = d \ln K_k = d \ln X_k = \gamma_k \Rightarrow \qquad d \ln C = \theta \gamma_k + (1-\theta) \gamma_c$.
To detrend divide capital and investment by X_{kt} and consumption by $X_{kt}^{\theta} X_{ct}^{1-\theta}$.
Detrended sequential problem:
$\max \Sigma_{t=0}^{\infty} \beta^t \left[\ln c_t + \phi \ln (1 - l_{ct} - l_{kt}) + (\theta \gamma_k + (1-\theta) \gamma_c) t\right] \qquad s.t. \qquad c_t \leq \widetilde{k}_{ct}^{\theta} (l_{ct})^{1-\theta}$
$i_{ct} + i_{kt} \leq \widetilde{k}_{kt}^{\theta} l_{kt}^{1-\theta} \qquad i_{ct} = \gamma_k \widetilde{k}_{ct+1} - (1-\delta) \widetilde{k}_{ct} \geq 0 \qquad i_{kt} = \gamma_k \widetilde{k}_{kt+1} - (1-\delta) \widetilde{k}_{kt} \geq 0$
Firm K problem: $\max_{\widetilde{k}_k, l_k} p(K_c, K_k) \widetilde{k}_k^{\theta} l_k^{1-\theta} - R_k(K_c, K_k) \widetilde{k}_k - W(K_c, K_k) l_k$
Firm C problem: $\max_{\widetilde{k}_c, l_c} \widetilde{k}_c^{\theta} l_c^{1-\theta} - R_c(K_c, K_k) \widetilde{k}_c - W(K_c, K_k) l_c$
Consumer problem: $v(k_c, k_k, K_c, K_k) = \max\limits_{c, k_c', k_k', l, x_c, x_k} \left[\ln c + \phi \ln(1-l) + \beta v(k_c', k_k', K_c', K_k')\right]$ s.t.
$c + p(K_c, K_k)(x_c + x_k) \leq W(K_c, K_k) l + R_c(K_c, K_k) k_c + R_k(K_c, K_k) k_k, \qquad K_k' = H_k(K_c, K_k),$
$x_c = \gamma_k k_c' - (1-\delta) k_c \geq 0, \qquad x_k = \gamma_k k_k' - (1-\delta) k_k \geq 0, \qquad K_c' = H_c(K_c, K_k).$
Recursive competitive equilibrium includes:
1) Value function $v(k_c, k_k, K_c, K_k)$
2) Decision rules for consumers, $k_c'(k_c, k_k, K_c, K_k)$, $k_k'(k_c, k_k, K_c, K_k)$, $c(k_c, k_k, K_c, K_k)$, $l(k_c, k_k, K_c, K_k)$, $x_c(k_c, k_k, K_c, K_k)$, $x_k(k_c, k_k, K_c, K_k)$
3) Decision rules for firm K: $l_k(K_c, K_k), \widetilde{k}_k(K_c, K_k)$
4) Decision rules for firm C: $l_c(K_c, K_k), \widetilde{k}_c(K_c, K_k)$
5) Price functions: $R_c(K_c, K_k)$, $R_k(K_c, K_k)$, $W(K_c, K_k), p(K_c, K_k)$
6) Laws of motion of aggregate capitals: $H_c(K_c, K_k)$, $H_k(K_c, K_k)$
s.t.
a) decision rules are solutions to agents' problems given prices
b) Markets clear given the state (K_c, K_k):
$\widetilde{k}_c = k_c = K_c, \qquad \widetilde{k}_k = k_k = K_k, \qquad l_c + l_k = l, \qquad c = K_c^{\theta} l_c^{1-\theta}, \qquad x_c + x_k = K_k^{\theta} l_k^{1-\theta}$
c) Perceptions are correct: $H_k(K_c, K_k) = k_k'(K_c, K_k, K_c, K_k)$, $H_c(K_c, K_k) = k_c'(K_c, K_k, K_c, K_k)$
Steady-state: $k_c, k_k, l_c, l_k, c, i, p, W, R_c, R_k$.

$$v(k_c, k_k, K_c, K_k) = \max_{k_c', k_k', l} \left[\begin{array}{c} \ln\{Wl + R_c k_c + R_k k_k - p(\gamma_k k_c' - (1-\delta) k_c + \gamma_k k_k' - (1-\delta) k_k)\} + \\ +\phi \ln(1-l) + \beta v(k_c', k_k', K_c', K_k') \end{array}\right]$$

FOC: $\frac{W}{c} = \frac{\phi}{1-l} \qquad \frac{p\gamma_k}{c} = \beta v_1'(k_c', k_k', K_c', K_k') = \beta v_2'(k_c', k_k', K_c', K_k')$
ENV: $v_1'(k_c, k_k, K_c, K_k) = \frac{R_c + (1-\delta)}{c} \qquad v_2'(k_c, k_k, K_c, K_k) = \frac{R_k + (1-\delta)}{c}$
Therefore, $v_1' = v_2' = v' = \frac{R_c + (1-\delta)}{c} = \frac{R_k + (1-\delta)}{c} = \frac{p\gamma_k}{\beta c}, \quad \frac{W}{c} = \frac{\phi}{1-l} \Rightarrow \boxed{R_c = R_k = \frac{p\gamma_k}{\beta} - (1-\delta)}$
Firms FOCs: $\theta p \frac{i}{k_k} = R_k, \qquad R_c = \theta \frac{c}{k_c}, \qquad (1-\theta) p \frac{i}{l_k} = W = (1-\theta) \frac{c}{l_c}$
$\Rightarrow \theta p \frac{i}{k_k} = \theta \frac{c}{k_c} = \frac{p\gamma_k}{\beta} - (1-\delta), \quad p\frac{i}{c} = \frac{k_k}{k_c} \qquad \Rightarrow \qquad \frac{W}{c} = \frac{\phi}{1-l} = \frac{1-\theta}{l_c} = \frac{(1-\theta)}{l_k} p \frac{i}{c} = \frac{(1-\theta)}{l_k} \frac{k_k}{k_c}$

Balances: $l_c + l_k = l$, $\boxed{c = k_c^\theta l_c^{1-\theta}}$, $\boxed{i = k_k^\theta l_k^{1-\theta}}$

S-S: $\frac{\phi}{1-l_c-l_k} = \frac{1-\theta}{l_c} = \frac{(1-\theta)}{l_k}\frac{k_k}{k_c} \Rightarrow l_c = \frac{1-\theta}{\phi+(1-\theta)}(1+l_k)$, $l_c\frac{k_k}{k_c} = l_k$

$\Rightarrow l_c = \frac{1-\theta}{\phi+(1-\theta)}\left(1 - l_c\frac{k_k}{k_c}\right) \Rightarrow \boxed{l_c = \frac{1}{\frac{\phi}{1-\theta}+1+\frac{k_k}{k_c}}}$, $\boxed{l_k = \frac{\frac{k_k}{k_c}}{\frac{\phi}{1-\theta}+1+\frac{k_k}{k_c}}}$

$\boxed{p} = \frac{c}{i}\frac{k_k}{k_c} = \frac{k_c^\theta l_c^{1-\theta}}{k_k^\theta l_k^{1-\theta}}\frac{k_k}{k_c} = \left(\frac{l_c}{l_k}\right)^{1-\theta}\left(\frac{k_c}{k_k}\right)^\theta \frac{k_k}{k_c} = \left(\frac{k_c}{k_k}\right)^{1-\theta}\left(\frac{k_c}{k_k}\right)^\theta\frac{k_k}{k_c} = \boxed{1}$

$[\gamma_k - (1-\delta)]\left(1+\frac{k_c}{k_k}\right) = \frac{i}{k_k} = \left(\frac{l_k}{k_k}\right)^{1-\theta} = \left(\frac{l_c}{k_c}\right)^{1-\theta} = \theta\frac{k_c^\theta l_c^{1-\theta}}{k_c}\frac{1}{\theta} = \frac{1}{\theta}\left[\frac{\gamma_k}{\beta} - (1-\delta)\right]$

$\boxed{\frac{k_c}{k_k} = \frac{1}{\theta}\left[\frac{\frac{\gamma_k}{\beta}-(1-\delta)}{\gamma_k-(1-\delta)}\right] - 1}$ $\boxed{k_k} = \boxed{\frac{\left(\frac{\theta}{\frac{\gamma_k}{\beta}-(1-\delta)}\right)^{\frac{1}{1-\theta}}}{\frac{\phi}{1-\theta}\left(\frac{1}{\theta}\left(\frac{\frac{\gamma_k}{\beta}-(1-\delta)}{\gamma_k-(1-\delta)}\right)-1\right)+\frac{1}{\theta}\left(\frac{\frac{\gamma_k}{\beta}-(1-\delta)}{\gamma_k-(1-\delta)}\right)}} = \frac{u^{\frac{1}{1-\theta}}}{\frac{\phi}{1-\theta}\left(\frac{1}{vu}-1\right)+\frac{1}{vu}}$

where $\frac{\theta}{\frac{\gamma_k}{\beta}-(1-\delta)} = u$ $\gamma_k - (1-\delta) = v$ $\boxed{\frac{W}{1-\theta}} = \frac{c}{l_c} = \left(\frac{k_c}{l_c}\right)^\theta = \boxed{\left(\frac{\theta}{\frac{\gamma_k}{\beta}-(1-\delta)}\right)^{\frac{\theta}{1-\theta}}}$

Calibration: $R + (1-\delta) = \frac{\gamma_k}{\beta} = \frac{6}{5}$, $\frac{i}{y} = \frac{i}{c+i} = \frac{1}{\frac{c}{i}+1} = \frac{1}{\frac{k_c}{k_k}+1} = \frac{9}{50} \Rightarrow \frac{k_c}{k_k} = \frac{41}{9}$

$\frac{1+\frac{k_k}{k_c}}{\frac{\phi}{1-\theta}+1+\frac{k_k}{k_c}} = \frac{1}{\frac{\phi}{1-\theta}\frac{41}{50}+1} = \frac{1}{3} \Rightarrow \frac{\phi}{1-\theta} = \frac{100}{41}$ $\frac{Rk_c+Rk_k}{Wl+Rk_c+Rk_k} = \frac{1}{1+\frac{W}{R}\frac{l}{k_c+k_k}} = \frac{1}{1+\frac{1-\theta}{\theta}} = \theta = \boxed{\frac{7}{20}} \Rightarrow$

$\phi = \frac{100}{41}\left(1-\frac{7}{20}\right) = \boxed{\frac{65}{41}} \Rightarrow \frac{k_c}{k_k} + 1 = \frac{1}{\theta}\left[\frac{\frac{\gamma_k}{\beta}-(1-\delta)}{\gamma_k-(1-\delta)}\right] = \frac{50}{9} \Rightarrow$

$\gamma_k - 1 + \delta = \frac{9}{50}\frac{20}{7}\left(\frac{1}{5}+\delta\right) = \frac{18}{35}\delta + \frac{18}{175} \Rightarrow \delta = \frac{193}{85} - \frac{35}{17}\gamma_k$

If $\gamma_k = \boxed{\frac{203}{200}}$, then $\beta = \gamma_k\frac{5}{6} = \boxed{\frac{203}{240}}$ $\delta = \frac{193}{85} - \frac{35}{17}\frac{203}{200} = \boxed{\frac{123}{680}}$

Transition process:

$\frac{W}{c} = \frac{\phi}{1-l}$ $\frac{p\gamma_k}{c} = \beta v_1'(k_c', k_k') = \beta v_2'(k_c', k_k')$ $v_1'(k_c, k_k) = \frac{R_c+(1-\delta)}{c}$ $v_2'(k_c, k_k) = \frac{R_k+(1-\delta)}{c}$

$\theta p\frac{i}{k_k} = R_k$, $R_c = \theta\frac{c}{k_c}$, $(1-\theta)p\frac{i}{l_k} = W = (1-\theta)\frac{c}{l_c}$ $l_c + l_k = l$, $c = k_c^\theta l_c^{1-\theta}$,

$i = k_k^\theta l_k^{1-\theta} = \gamma_k k_c' - (1-\delta)k_c + \gamma_k k_k' - (1-\delta)k_k$

Derive: $W = \frac{c\phi}{1-l_c-l_k} = (1-\theta)p\frac{i}{l_k} = (1-\theta)\frac{c}{l_c}$ $v_2' = v_1'$ $R_k = R_c = \theta p\frac{i}{k_k} = \theta\frac{c}{k_c}$

$p = \frac{c}{i}\frac{k_k}{k_c} = \frac{c}{i}\frac{l_k}{l_c}$ $l_k = l_c\frac{k_k}{k_c}$ $\frac{\phi}{1-\theta}l_c = 1 - l_c - l_c\frac{k_k}{k_c}$ $l_c = \frac{1}{\frac{\phi}{1-\theta}+1+\frac{k_k}{k_c}}$ $l_k = \frac{\frac{k_k}{k_c}}{\frac{\phi}{1-\theta}+1+\frac{k_k}{k_c}}$

$p\frac{\gamma_k}{c} = \beta v_1'(k_c', k_k') = \beta\frac{R_c'+(1-\delta)}{c'} = \beta\frac{\theta\frac{c'}{k_c'}+(1-\delta)}{c'}$ $\frac{1}{k_k^\theta(l_k)^{1-\theta}}\frac{k_k}{k_c}\gamma_k = \beta\left(\theta + (1-\delta)\frac{k_c'}{k_c'^\theta(l_c')^{1-\theta}}\right)\frac{1}{k_c'}$

$\left(\left(\frac{\phi}{1-\theta}+1\right)k_c + k_k\right)^{1-\theta}\frac{1}{k_c}\gamma_k = \beta\left(\theta + (1-\delta)\left(\left(\frac{\phi}{1-\theta}+1\right)k_c' + k_k'\right)^{1-\theta}\right)\frac{1}{k_c'}$

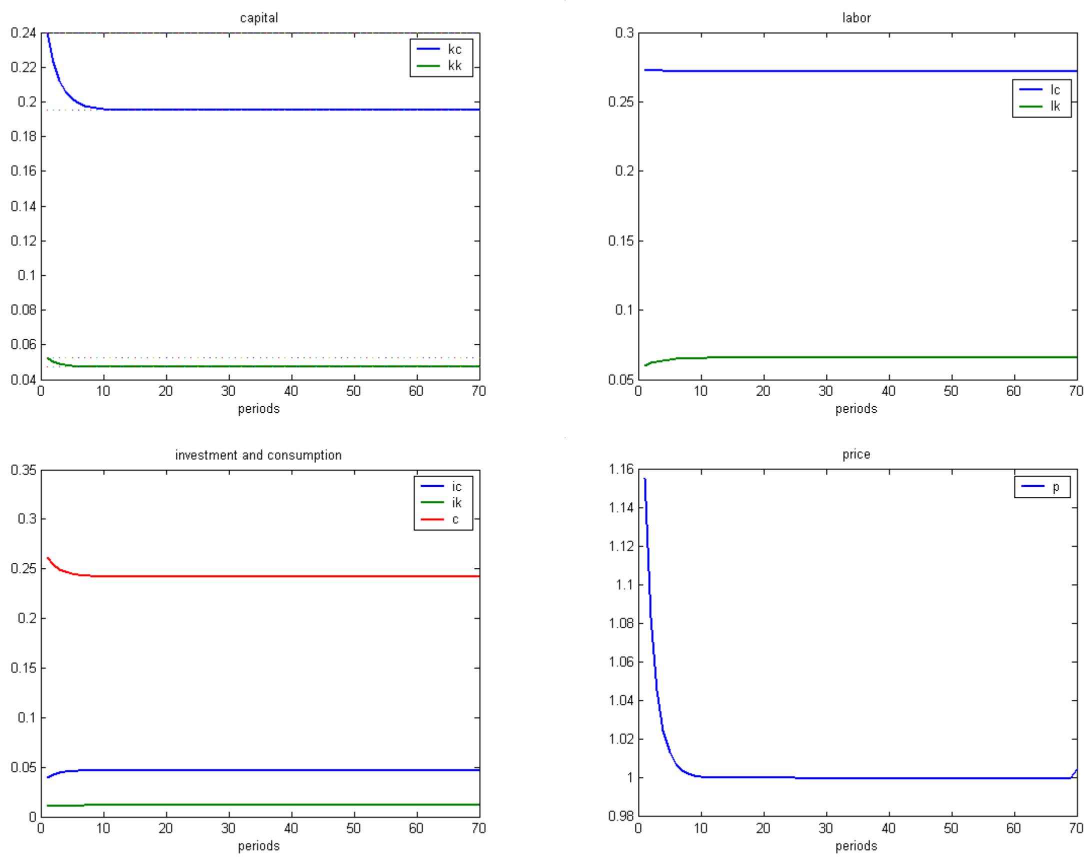
$\boxed{(ak_c + k_k)^{1-\theta}\frac{1}{k_c}\gamma_k = \beta\left(\theta + b(ak_c' + k_k')^{1-\theta}\right)\frac{1}{k_c'}}$ Denote: $\left[\frac{\phi}{1-\theta}+1\right] = a$, $1-\delta = b$

$\gamma_k k_c' - (1-\delta)k_c + \gamma_k k_k' - (1-\delta)k_k = k_k^\theta l_k^{1-\theta} = k_k^\theta\left(\frac{\frac{k_k}{k_c}}{\frac{\phi}{1-\theta}+1+\frac{k_k}{k_c}}\right)^{1-\theta} = \frac{k_k}{(ak_c+k_k)^{1-\theta}}$

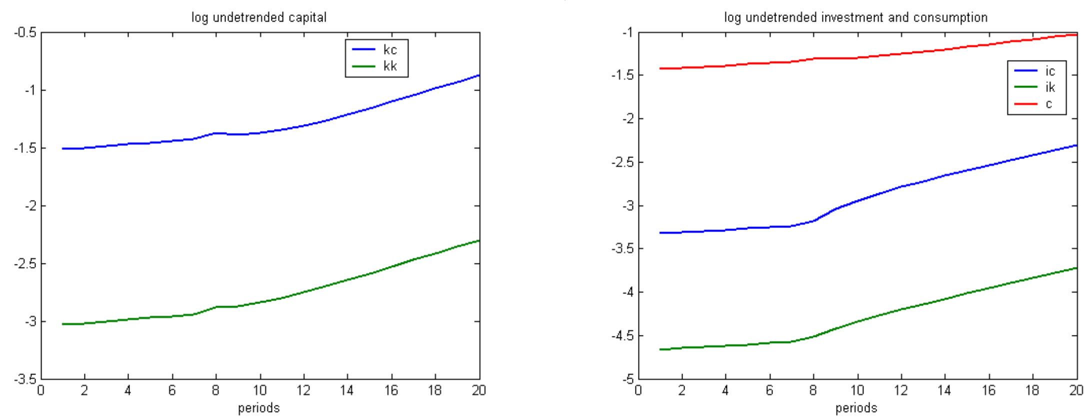
$\boxed{\gamma_k(k_c' + k_k') = (1-\delta)(k_c + k_k) + \frac{k_k}{(ak_c+k_k)^{1-\theta}}}$

The dynamic system: $\left\{\frac{\beta}{\gamma_k}\frac{k_c}{(ak_c+k_k)^{1-\theta}} = \frac{k_c'}{\theta+b\left(ak_c'+k_k'\right)^{1-\theta}}, \frac{k_k}{(ak_c+k_k)^{1-\theta}} + b(k_k+k_c) = \gamma_k(k_c'+k_k')\right\}$

We solve for the transition process from the initial steady-state to the new one when $\gamma_k = \frac{212}{200}$.



Intuition: Return to capital increases in the investment sector. This is reflected in the price of investment goods, which jumps up, and then gradually goes back. There is a labor shift from consumption to investment, and the total time the guys work increases, because the growth rate has increased and they need to sustain it with higher investment and consumption growth. Investment in both sectors bursts out, capital and consumption in both sectors temporarily slow down in transition to a new balanced growth path, which is faster.



Exercise 2

$$\max E\Sigma\beta^t\left[\ln c_t+\phi\left(1-l\right)\right] \qquad c_t+k_{t+1}-\left(1-\delta\right)k_t\leq A_tk_t^{\theta}l_t^{1-\theta} \qquad 0\leq l\leq 1$$

$$\max \Sigma\beta^t\left[\ln\left(A_tk_t^{\theta}l_t^{1-\theta}+\left(1-\delta\right)k_t-k_{t+1}\right)+\phi\left(1-l_t\right)\right]$$

FOC: $\frac{1}{c_t}\frac{A_tk_t^{\theta}l_t^{1-\theta}}{l_t}\left(1-\theta\right)=\phi \qquad \frac{1}{c_t}=\beta\frac{1}{c_{t+1}}\left(\theta\frac{A_{t+1}k_{t+1}^{\theta}l_{t+1}^{1-\theta}}{k_{t+1}}+\left(1-\delta\right)\right)$

$Ak^{\theta}l^{1-\theta}-Wl-Rk\rightarrow\max_{k,l} \qquad Wl=\left(1-\theta\right)Ak^{\theta}l^{1-\theta} \qquad \theta=\boxed{\frac{1}{3}}$

Normalize labor to 1 in period 1: t=1: $\phi=\frac{1}{c}\frac{y}{l}\left(1-\theta\right)=\frac{2}{3}\frac{4}{3}=\boxed{\frac{8}{9}}$

$$A\left(\frac{k}{l}\right)^{\theta}=\frac{\phi}{(1-\theta)}c \qquad 1=\beta\left(\theta A\left(\frac{l}{k}\right)^{1-\theta}+\left(1-\delta\right)\right) \qquad c+\delta k=Ak^{\theta}l^{1-\theta}$$

$$\delta=A\left(\frac{l}{k}\right)^{1-\theta}\left[1-\frac{1}{l}\frac{(1-\theta)}{\phi}\right] \qquad \left[1-\frac{1}{l}\frac{(1-\theta)}{\phi}\right]=\frac{\theta}{\frac{1}{\beta}-(1-\delta)}\delta \qquad l=\frac{1}{1-\frac{\theta\delta}{\frac{1}{\beta}-(1-\delta)}}\frac{(1-\theta)}{\phi}$$

$$R=\theta A\left(\frac{l}{k}\right)^{1-\theta}+1-\delta=\frac{1}{\beta}=\frac{53}{50} \qquad \beta=\boxed{\frac{50}{53}}$$

$$\frac{i}{y}=1-\frac{c}{y}=1-\frac{1}{l}\frac{(1-\theta)}{\phi}=\frac{\theta\delta}{\frac{1}{\beta}-(1-\delta)}=\frac{\frac{1}{3}\delta}{\frac{53}{50}-(1-\delta)}=\frac{1}{4} \qquad \delta=3\left(\frac{53}{50}-\left(1-\delta\right)\right)-3\delta=\boxed{\frac{9}{50}}$$

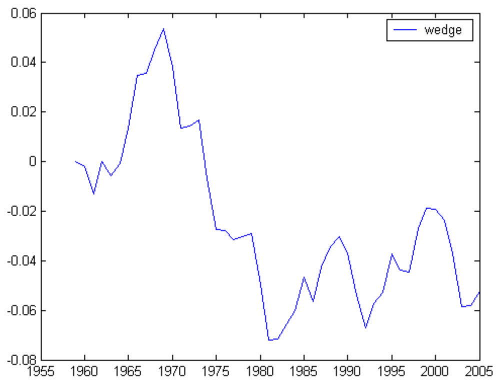
Wedge: $\psi_t=\frac{\phi}{(1-\theta)}\frac{c_t}{y_t}l_t=\frac{c_t}{y_t}\frac{l_t}{l_1}\frac{y_1}{c_1} \qquad \frac{\phi}{(1-\theta)}\frac{c_1}{y_1}l_1=1$

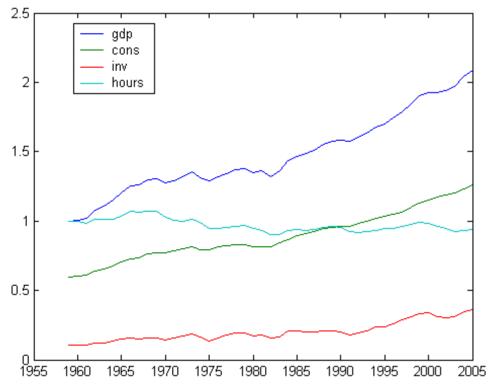
If there was a labor tax, than the problem would look like:

$$\max E\Sigma\beta^t\left[\ln c_t+\phi\left(1-l\right)\right] \qquad c_t+k_{t+1}-\left(1-\delta\right)k_t\leq\left(1-\tau_t\right)w_tl_t+r_tk_t \qquad 0\leq l\leq 1$$

$$Ak^{\theta}l^{1-\theta}-wl-rk\rightarrow\max_{k,l} \qquad \forall t$$

Then, the FOC for labor looks like this: $\frac{1}{c_t}\left(1-\tau_t\right)\frac{A_tk_t^{\theta}l_t^{1-\theta}}{l_t}\left(1-\theta\right)=\phi \;\Rightarrow\; \psi_t=\left(1-\tau_t\right)$





Linearization: (1) $w_t\frac{1}{c_t}\frac{Ak_t^{\theta}l_t^{1-\theta}}{l_t}\left(1-\theta\right)=\phi \qquad \boxed{\widetilde{w}_t-\widetilde{c}_t+\theta\widetilde{k}_t-\theta\widetilde{l}_t=0}$

(2) $\frac{1}{c_t}=\beta E\frac{1}{c_{t+1}}\left(\theta\frac{Ak_{t+1}^{\theta}l_{t+1}^{1-\theta}}{k_{t+1}}+\left(1-\delta\right)\right) \qquad E\widetilde{c}_{t+1}-\widetilde{c}_t=\frac{\theta A\left(\frac{l}{k}\right)^{1-\theta}}{\theta A\left(\frac{l}{k}\right)^{1-\theta}+(1-\delta)}\left(1-\theta\right)E\left(\widetilde{l}_{t+1}-\widetilde{k}_{t+1}\right)$

$\frac{1}{\beta}-\left(1-\delta\right)=\theta A\left(\frac{l}{k}\right)^{1-\theta} \qquad \boxed{E\widetilde{c}_{t+1}-\widetilde{c}_t=\left(1-\beta\left(1-\delta\right)\right)\left(1-\theta\right)\left(E\widetilde{l}_{t+1}-\widetilde{k}_{t+1}\right)}$

(3) $c_t+k_{t+1}-\left(1-\delta\right)k_t\leq A_tk_t^{\theta}l_t^{1-\theta} \qquad \frac{c}{y}\widetilde{c}_t+\frac{k}{y}\widetilde{k}_{t+1}-\left(1-\delta\right)\frac{k}{y}\widetilde{k}_t=\theta\widetilde{k}_t+\left(1-\theta\right)\widetilde{l}_t$

$\boxed{\frac{c}{y}\widetilde{c}_t+\frac{k}{y}\widetilde{k}_{t+1}-\left(1-\delta\right)\frac{k}{y}\widetilde{k}_t=\theta\widetilde{k}_t+\left(1-\theta\right)\widetilde{l}_t} \qquad \rho=0.9744$

$$\frac{c}{y}=1-\frac{\theta\delta}{\frac{1}{\beta}-(1-\delta)}=\frac{3}{4} \qquad \frac{c}{y}+\delta\frac{k}{y}=1 \qquad \frac{k}{y}=\frac{1}{\delta}\left(1-\frac{c}{y}\right)=\frac{50}{36}$$

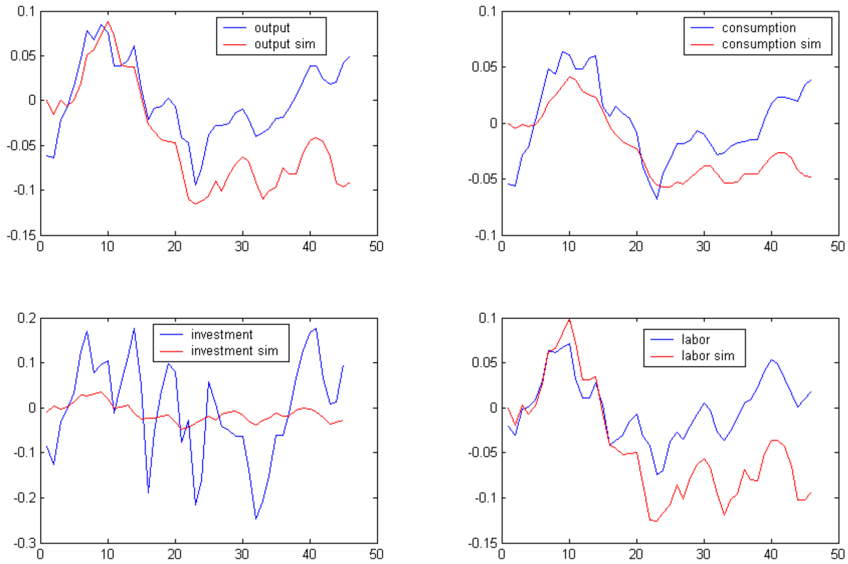
(1-4): $\widetilde{w}_t-\widetilde{c}_t+\frac{1}{3}\widetilde{k}_t-\frac{1}{3}\widetilde{l}_t=0 \qquad E\widetilde{c}_{t+1}-\frac{8}{53}E\widetilde{l}_{t+1}+\frac{8}{53}\widetilde{k}_{t+1}=\widetilde{c}_t$

$\frac{3}{4}\widetilde{c}_t+\frac{50}{36}\widetilde{k}_{t+1}-\frac{41}{36}\widetilde{k}_t=\frac{2}{3}\widetilde{l}_t \qquad \widetilde{w}_{t+1}=\rho\widetilde{w}_t+\varepsilon_t$

$$\begin{bmatrix} 0 & -1 & -\frac{1}{3} & 1 & 0 & 0 \\ \frac{8}{53} & -1 & 0 & 0 & 1 & -\frac{8}{53} \\ \frac{50}{36} & \frac{3}{4} & -\frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_{t+1} \\ c_t \\ l_t \\ w_t \\ Ec_{t+1} \\ El_{t+1} \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{41}{36} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_t \\ c_{t-1} \\ l_{t-1} \\ w_{t-1} \\ Ec_t \\ El_t \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \varepsilon_t + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_t \\ v_t \end{bmatrix}$$

Use gensys and calculate: $\widetilde{y}_t = \theta \widetilde{k}_t + (1-\theta)\widetilde{l}_t \qquad \widetilde{i}_t = \frac{k}{y}\widetilde{k}_{t+1} - (1-\delta)\frac{k}{y}\widetilde{k}_t$



Correlation matrices:

data	c	l	y	i
c	1	0.77	0.95	0.70
l		1	0.90	0.75
y			1	0.81
i				1

model	c	l	y	i
c	1	0.96	0.98	0.68
l		1	0.99	0.81
y			1	0.77
i				1

The apparent problem here is almost no volatility in investment, while in the data investment is much more volatile, than all other variables. The model generates a bit too high correlations for consumption, labor and output and a bit too low ones for investment. We obviously lack productivity shocks here, which would create volatility and correlations for investment.

Sources of data: BEA, BLS.