Final Exam: Econ 202C

June 9-13, 2006

There are 5 problems. The exam is open-books, open notes open everything, all sources of information are acceptable, except your classmates. To receive maximum credit, you need to complete at least 4 questions.

Problem 1: Money and Capital

Time is discrete and infinite. There is a single consumption good ("corn"), which can either be consumed, or planted in each period, to yield a crop in the following period. There are two types of consumers, odd and even. Even types have high crop yields in even periods, but low yields in odd periods, while odd types have high crop yields in odd periods, and low yields in even periods. These yields are summarized below, where I denote the yields for k units of corn planted in $t-1$, for even and odd periods and types:

$t =$	*even*	*odd*	*even*	*odd*	...
odd	$\underline{A}f(k)$	$\overline{A}f(k)$	$\underline{A}f(k)$	$\overline{A}f(k)$	...
even	$\overline{A}f(k)$	$\underline{A}f(k)$	$\overline{A}f(k)$	$\underline{A}f(k)$	...

where $f(k)$ is increasing, and concave, and $f(0) = 0$, $f'(0) = \infty$ and $\lim_{k\to\infty} f'(k) = 0$, and $\overline{A} > \underline{A}$. Capital fully depreciates each period. At date 0, even types start with an initial endowment of y units of corn. For both types, preferences over consumption allocations are given by $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where u is increasing and concave, $u'(0) = \infty$, $\lim_{c\to\infty} u'(c) = 0$, and $\beta \in (0,1)$.

(a) Define the social planner problem for this economy. Characterize the first-order conditions for efficient optimal allocations and investment. You may assume that the planner assigns equal weight to agents of both types.

(b) Suppose next that in each period, agents can engage in trade of one-period uncontingent bonds. These bonds are fully enforcible. Show that this trading arrangement can implement the social planner's solution, and derive the associated sequence of bond prices and period by period bond holdings (as a function of the optimal allocations).

(c) Monetary trade: suppose now that at date 0, odd types hold M units of money each, and that in each period, agents can trade money for corn. Define a monetary equilibrium of this economy, and derive the relevant conditions that characterize equilibrium allocations.

(d) Characterize the conditions for a steady state equilibrium in which the value of money remains constant over time.

(e) Compare the steady-state monetary equilibrium and the planner's solution.

Problem 2: A Cash-in-Advance model

Consider the following Cash-in-Advance economy: a representative household maximizes

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_{1t}, c_{2t}\right) - v\left(L_t\right) \right]$$

where c_{1t} and c_{2t} denote the consumption of two different goods at date t, and L_t the household's labor supply. There is a representative producer for each good who, in each period rents capital k_t and hires labor l_t from the representative household to produce his good, according to a constant returns to scale technology $y_{it} = f\left(k, l\right)$. Capital depreciates at a rate δ. The household has to use money balances held at the begining of each period to purchase good 1, but can buy good 2 on credit, and pay in Cash at the end of the period, and only the credit good can be stored to serve as capital during the next period. At the beginning of each period, the household receives a lump sum transfer (or pays a tax) of μM_{t-1}, where M_{t-1} denote the aggregate money supply carried forward from the previous period.

(a) Define the competitive equilibrium for this economy. Make sure to properly specify the Cash-in-Advance constraint for good 1.

(b) Derive the relevant first-order conditions for this economy.

(c) Solve these equations for the steady-state levels of l, c_1, c_2, the rate of inflation π and real balances m as a function of μ.

(d) What effects do changes in μ have on the steady-state output level? What is the optimal rate of money growth?

Problem 3: A search model of money with specialization

Consider the following search model of money: There is a large number of traders. In each period, a trader can either be a "commodity trader" and hold a commodity, or a "money trader" and hold one unit of money. There are M units of money in the economy. Commodities are 'specialized', and we let x stand for the degree of a commodity's specialization, which is the probability that a randomly selected agent in this economy likes a particular good, and is willing to accept it for consumption.

In each period, all traders are randomly matched into pairs, and trade occurs, if both parties agree to it. At the end of each period, a trader consumes whenever he has acquired a good that he is willing to consume. After consuming, a trader then produces a new good with a degree of specialization x, where we let $0 \leq x \leq \frac{1}{2}$ be the producer's choice variable. We let $c(x)$ denote the cost of producing a good with specialization x, and suppose $c(x)$ is increasing and convex, and $c(0) = c'(0) = 0$: goods that are more generally acceptable are more expensive to produce. Finally, there is an instantaneous utility U, whenever consumption takes place, and a discount rate $\beta < 1$.

Traders therefore must choose the degree of specialization x, when they produce, and their willingness to accept money in a transaction (we assume that they are always willing to accept a good that they are willing to consume).

(a) Formally define the trader's strategies and the steady-state equilibrium of this economy.

(b) Let X denote the average degree of specialization for commodity traders in this economy. Taking X and Π as given, write down the set of Bellman equations for the trader's optimization problem, denoting by V_m, and $V_c(x)$ the value of being a money trader, and a commodity trader with a commodity with a degree of specialization x.

(c) Suppose for the moment that x is fixed. What values of Π are sustainable in equilibrium?

(d) For each equilibrium value of Π, characterize a trader's optimal specialization choice. How does it depend on X and on M? How does the equilibrium value of x depend on M? If there are multiple equilibria, compare the degree of specialization across equilibria.

(e) Consider instead the problem of a utilitarian social planner who seeks to maximize the average welfare level of the agents. What steady-state values of X and Π would the planner find optimal? How do they compare to the equilibrium?

Problem 4: A static incomplete information problem

There are two islands, A and B. In each island there is a large number of producers, which produce a single homogeneous good. The demand for the goods from islands A and B is given by

$$c_A = \frac{\alpha M}{p_A} \text{ and } c_B = \frac{(1-\alpha)\, M}{p_B}$$

where M measures total nominal spending on the two goods, and α is the expenditure share on good A. We assume that both M and α are stochastic, and can take on one of two values each: $M \in \left\{\underline{M}, \overline{M}\right\}$ and $\alpha \in \{\underline{\alpha}, \overline{\alpha}\}$, with $\underline{\alpha} < 1/2$, $\overline{\alpha} = 1 - \underline{\alpha}$, $\underline{M} < \overline{M}$ and $\underline{\alpha}\overline{M} = \overline{\alpha}\underline{M}$. All four pairs of (α, M) are equally likely ex ante.

The producers' objective is given by the following profit function:

$$\Pi = \frac{p_i q_i}{M} - \frac{1}{2} q_i^2,$$

where q_i denotes their output, and p_i the market price, for the good on island $i \in \{A, B\}$. This function has the following interpretation: the first term measures nominal revenue. As a real measure of revenues, this nominal revenue is divided by total nominal spending in the economy, M. The second term measures a real disutility of effort due to providing the output. (We could be deriving this profit structure and the spending on goods from more primitive preference/technology assumptions, but I will skip that for now).

When producers set their quantity, they see the market price for their island's good, but not the other price, nor can they directly observe the pair (α, M). They set q to maximize their expected profits.

(a) Before going to the incomplete information model, suppose first that all agents see not only the price on their own island, but also the entire pair (α, M). Characterize the producers' optimal supply decisions, and the resulting market-clearing prices and quantities. How do they depend on M and α.

(b) Now, go to the private information economy. Define the equilibrium concept for this economy.

(c) Characterize the equilibrium with private information (Hint: you may need to start from a judicious guess about what information p ought to reveal in equilibrium, then characterize allocations based on that guess, and finally verify whether your guess was correct...).

(d) Compare the equilibrium allocations with complete and incomplete information. How does it compare to the one we saw in class?

Problem 5: A one-time money injection

Consider the following version of the Calvo-style sticky price model with monopolistic competition. Time is discrete and infinite. There is a single final good, and a measure 1 continuum of intermediate goods. There is a final goods sector, which uses the intermediates to produce the final good according to the technology

$$Y_t = \left[\int_0^1 \left(y_t^i \right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

The intermediate producers are monopolists, and can set the nominal prices at which they sell their good. In each period, they hire labor at a constant rate M to produce the demand for the final good. Their technology is given by the production function $y = n^{\alpha}$, with $\alpha \leq 1$. Finally, there is a representative household that spends a constant nominal amount M on the final good in each period.

(a) For a given value of Y_t, determine the demand for intermediates, and define the profit function for the intermediate firms, as a function of their own prices, the final goods price and the total demand.

(b) Characterize the flexible price equilibrium, in which all firms can adjust their prices in every period.

From now on, suppose that in each period, a fraction $\lambda \in (0, 1)$ can adjust their prices in any given period, and that the opportunity to adjust does not depend on when a firm last adjusted its prices.

(c) Formulate the new optimization problem for the intermediate firms, taking into account that they adjust only infrequently. Define the recursive competitive equilibrium with Calvo-sticky prices, and derive the non-linear system of equations for (p_t, P_t) as a function of P_{t-1} and M (which is held constant) that characterizes equilibrium price adjustment.

(d) Suppose that the initial price level is at the flexible price equilibrium. Show that this is also a steady-state equilibrium of the sticky price economy.

(e) Suppose now that in some period, unexpectedly the money supply changes from M to M' and stays there in all future periods. This is known to the firms at the time of the shock. Assuming that the initial price level was at the original flexible price equilibrium, derive the conditions that describe the price adjustment dynamics. Using an approximation similar to the one derived in class, show that prices must converge to the new flexible price equilibrium.