**Econ 202b Homework 3. Harold Cole.**

**Problem 11:** We are requiring that the individual almost surely (a.s.) be able to payoff his debts. This seems like a very different requirement than the present-value budget constraint that we impose in complete market models in which we merely require that the expect present value of expenditures not exceed the expected present value of income and initial wealth. Try and reconcile these two conditions by showing that the complete markets present value budget constraint implies the budget constraint we have imposed in our incomplete markets model.

Consider a model with complete Arrow securities in which $a_{t+1}(h_{t+1}|h^t)$ denotes that number of real claims acquired in period $t$ with history $h^t$ which pays off one unit in period $t+1$ if the realization is $h_{t+1}$ (in which case the history becomes $h^{t+1} = (h_{t+1}, h^t)$). Assume that these securities trade at price $q_t(h_{t+1}|h^t)$ in period $t$ and payoff one real unit per claim. Assume that individual's income is stochastic and given by $y(h^t)$.

*Hint:* With compete markets, we can freely assign excess net income (income over expenditures) across different realizations of the history $h^t$.

**Problem 12:** In a famous paper, Lucas consider the equilibrium of an economy in which money was the only asset, and in which an agent receives a constant income $y$ in each period, and faces stochastic preference shocks. These shocks, $z$, can be assume to be uniformly distributed upon a closed interval $[a, b]$. The agent's preferences are given by

$$E\left[\sum_{t=0}^{\infty} \beta^t U(c_t, z_t),\right] \qquad \text{where } U(c,z) = z\frac{[c^{1-\mu}-1]}{1-\mu}.$$

The agent's problem can be written as

$$v(m,z) = \max\left[U(c,z) + \beta\int v(m',z')\mu(dz')\right]$$

subject to (1) $m' + c - m - y \leq 0.$ (2) $c - m \leq 0.$

If you're feeling bold, try to do exercises 13.5 in Lucas and Stokey. However, compute the solution to the agent's problem in terms of his optimal money accumulation policy $M(m, z)$ assuming that $z = [.5, 1, 1.5]$ with equal probability, and that $\beta = .9$. Simulate the economy as in Aiyagari above (drawing 10,000, etc.). Note that you don't have to iterate here since you already now the rate of return in a stationary economy in which the price level is constant. Compute the mean and coefficient of variation of the agent's money holdings in this economy.

**Problem 13.** Extra credit will be given to those who replicate Aiyagari's result.

Assume that $F(K,L) = K^{\alpha}L^{1-\alpha}$ $K_{t+1} = (1-\delta)K_t + X_t$

$$u(c) = \frac{[c^{1-\mu}-1]}{1-\mu} \qquad \beta = .96 \quad \alpha = .36 \quad \delta = 0.08 \quad \mu \in [1,3,5]$$

and that $l_t$ is distributed according to $\log(l_t) = \rho\log(l_{t-1}) + \sigma(1-\rho^2)^{1/2}\varepsilon_t$

$$\varepsilon_t \sim N(0,1) \qquad \sigma \in \{0.2, 0.4\} \quad \rho = [0, 0.3, 0.6, 0.9]$$

- As $\rho$ and $\sigma$ rise, the net return to capital goes from 4.16 to $-0.346$ %. This compares with the complete markets return of $\beta^{-1} = 4.17$ %.
- The aggregate savings rate as a percentage of income goes from 23.87% to 37.63%.