Econ202B Homework #3 Anton Cheremukhin

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Exercise 1 (11)

The flow budget constraint without uncertainty: $c_t + a_{t+1} = wl_t + (1+r)\,a_t$
Iterating it forward we get: $\sum_{t=0}^{T} \frac{c_t - wl_t}{(1+r)^t} = (1+r)\,a_0 + \frac{a_T}{(1+r)^T}$
In case of uncertainty $c_t^i + \sum_{j=1}^{S} q_t\left(h_{t+1}^j|h_t^i\right) a_{t+1}^j\left(h_{t+1}^j|h_t^i\right) = wl_t^i + (1+r)\,a_t\left(h_t^i|h_{t-1}\right)$

According to Arrow a full set of state-contingent claims which pay one unit next period in a specific state of nature is enough to implement complete markets, so we can represent all asset holdings in time-zero prices: $q_{t+1}^0\left(s_{t+1}^j\right) = \sum_{i=1}^{S} q_t^0\left(s_t^i\right) q_t\left(h_{t+1}^j|h_t^i\right)$. Iterating forward we get:

$\sum_{t=0}^{T}\sum_{j=1}^{S} q_t^0\left(s_t^j\right)\frac{c_t^j - wl_t^j}{(1+r)^t} = (1+r)\,a_0 + \sum_{j=1}^{S} q_T^0\left(s_T^j\right)\frac{a_T\left(s_t^j\right)}{(1+r)^T}$

In the complete markets model we impose the no-ponzi-game condition: $\lim_{T\to\infty}\frac{a_T\left(s_t^j\right)}{(1+r)^T} \geq 0$ for each state. It means that no agent may die in debt in any state (which is captured by the word "almost surely"). In the incomplete market model we impose the same restriction. Hence, in both cases we have the following identical lifetime budget constraint:

$\sum_{t=0}^{\infty}\frac{c_t - wl_t}{(1+r)^t} = (1+r)\,a$ $\qquad\sum_{t=0}^{T}\sum_{j=1}^{S} q_t^0\left(s_t^j\right)\frac{c_t^j - wl_t^j}{(1+r)^t} = (1+r)\,a_0$

Exercise 2 (12) *Stokey-Lucas 13.5*

$v(m,z) = \max_{c,m'\geq 0}\left[U(c,z) + \beta\int v(m',z')\,\mu(dz')\right]$ s.t. $c - m \leq y - m'$, $c - m \leq 0$

(a) Define $F(m,m') = U(m+y-m')$ and $\Gamma(m) = [y, m+y]$

Since $X = R_+$ and the shock preferences are i.i.d. X is a convex Borel set and the transition operator satisfies the Feller property. Since $y \in \Gamma(m)$ and $m \in X$, $\Gamma(m)$ is a closed interval defined by continuous functions of m, and continuity and boundedness of utility implies same properties of F, by theorem 9.6 there is a unique fixed point and the value function is bounded and continuous and the policy correspondence is non-empty, compact-valued and u.h.c.

(b) Since U is strictly increasing and the upper bound of $\Gamma(m)$ is strictly increasing in m by theorem 9.7 the value function is strictly increasing. The value function is concave since:

$v(\lambda m_0 + (1-\lambda)\,m_1) = E\left[\sum_{t=0}^{\infty}\beta^t U\left(c_t^{\lambda}, z_t\right)\right] \geq E\left[\sum_{t=0}^{\infty}\beta^t U(\lambda c_{t0} + (1-\lambda)\,c_{t1}, z_t)\right] >$
$E\left[\sum_{t=0}^{\infty}\beta^t\left\{\lambda U(c_{t0}, z_t) + (1-\lambda)\,U(c_{t1}, z_t)\right\}\right] = \lambda v(m_0) + (1-\lambda)\,v(m_1)$
$v_m'(m,z) = U_c'(m+y-m')$, if $m' = y$ then
$v_m'(m,z) = \left[U(m,z) + \beta\int v(y,z')\,\mu(dz')\right]_m' = U_c'(m,z)$

Hence, $v()$ is differentiable and due to concavity continuously differentiable.

(c) Since $v()$ is differentiable and $\Gamma(m)$ is convex by theorem 9.8 the policy function is single-valued and continuous.

(d) FOC: $U_m'(m+y-g(m,z), z) \geq \int v_m'(g(m,z), z')\,\mu(dz')$ with $=$ if $m' > y$.
For the unconstrained problem $w(m,z) = \max_{m+y\geq m'\geq 0}\left[U(m+y-m',z) + \beta\int w(m',z')\,\mu(dz')\right]$
FOC: $U_m'(m+y-h(m,z), z) = \int w_m'(h(m,z), z')\,\mu(dz')$.
If $h(m,z)$ is non-increasing in m than $w(h(m,z), z')$ by concavity of $w()$ is increasing in m.
Hence, $m+y-h(m,z)$ is increasing in m which leads to a contradiction. Hence, $h(m,z)$ has to be increasing. Therefore there exists $\phi(z)$ such that $h(\phi(z), z) = y$. From the restricted problem we can see that $g(m,z) = \max\{h(m,z), y\} = \{y \text{ for } m < \phi(z) \text{ and } h(m,z) \text{ otherwise}\}$.

(e) Since $U(m,z)$ and $v(m,z)$ are concave and $g(m,z)$ is increasing from the first-order condition it follows that $c(m,z)$ is also increasing in m.

(f) In equation $U'_m(y,z) \geq \int v'_m(\bar{m},z')\,\mu(dz')$ l.h.s. is constant and r.h.s. is increasing in $\bar{m}$.
By concavity and boundedness of $v()$: $0 \leq v'_m(m,z)\,m \leq v(m,z) - v(0,z) \leq B - v(0,z)$
Hence, $\lim_{m\to\infty} v'_m(m,z) = 0$. Besides, $U'_m(y,\underline{z}) < \int v'_m(\bar{m},z')\,\mu(dz')$.
Hence, $\bar{m}$ exists and is unique.

(g) The result follows from FOC and ENV: $U'_m(y,\bar{z}) > \int v'_m(y,z')\,\mu(dz') = \int U'_m(y,z')\,\mu(dz')$.

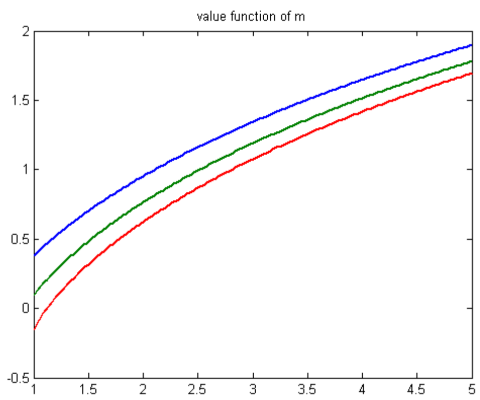
(h) By concavity of U and FOC $g(m,z)$ is weakly decreasing in z.

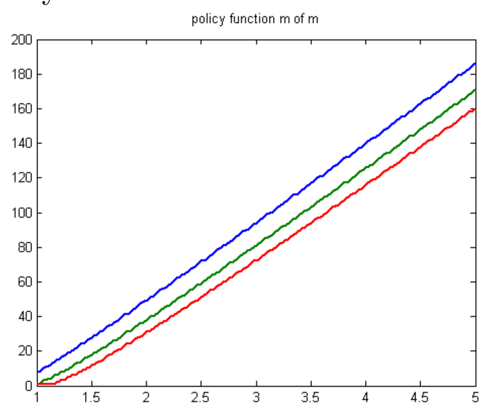
(i) $P(m,A) = \int 1_{\{g^{-1}(m,A)\}}\mu(dz') = \mu[\{z : g(m,z) \in A\}]$ satisfies Feller property.
If $\phi^{-1}(y) < \bar{z}$ than the mixing condition is fulfilled, which guarantees strong convergence to a unique invariant distribution.

(j) Defining $f(m) = m$ we get: $\int\int g(m,z)\,\mu(dz')\,\lambda^*(dm) = \int\int f(m')\,P(m,dm')\,\lambda^*(dm) = f\langle Tf,\lambda^*\rangle = \langle f,T^*\lambda^*\rangle = \langle f,\lambda^*\rangle = M/p$

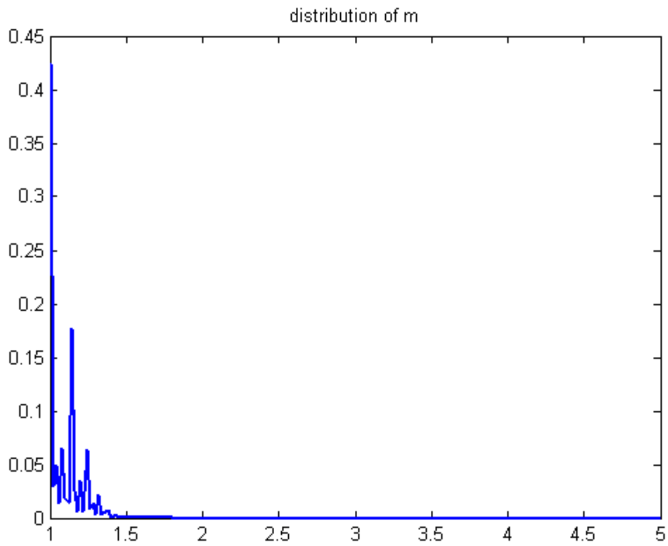
Numerical solution for $U(c,z)$ CRRA with $\mu = 3$, $\beta = 0.9$, $y = 1$, $z \in \{0.5; 1; 1.5\}$ i.i.d.:
Resulting $E[m] = 1.0894$ $\quad Var[m] = 0.0106$

Value Functions and Policy Functions:





Stationary Distribution:



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Exercise 3 (13) *Ayagari*

Constant parameters: $\beta = 1/(1+\nu)$ $\nu = 0.0416$ $\alpha = 0.36$ $\delta = 0.08$ $b = 0$

Results:

σ_ε	μ	ρ	K	r	w	s	$V[a]$
0.2	1	0.9	5.52	0.0406	1.184	0.2388	3.704
0.2	1	0.6	5.52	0.0406	1.184	0.2388	3.464
0.2	1	0.3	5.51	0.0408	1.183	0.2384	4.206
0.2	1	0	5.51	0.0408	1.183	0.2384	4.734

σ_ε	μ	ρ	K	r	w	s	$V[a]$
0.2	2	0.9	5.68	0.0385	1.196	0.2431	2.410
0.2	2	0.6	5.60	0.0395	1.190	0.2409	3.846
0.2	2	0.3	5.57	0.0399	1.188	0.2402	4.032
0.2	2	0	5.56	0.0401	1.187	0.2399	4.625

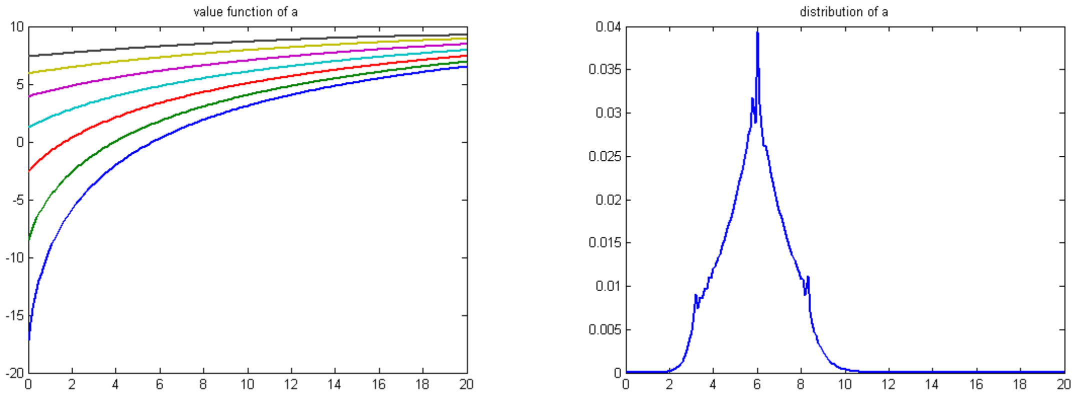
σ_ε	μ	ρ	K	r	w	s	$V[a]$
0.2	3	0.9	5.88	0.0359	1.211	0.2486	2.019
0.2	3	0.6	5.69	0.0383	1.197	0.2434	3.635
0.2	3	0.3	5.64	0.0390	1.193	0.2420	4.051
0.2	3	0	5.61	0.0394	1.191	0.2413	5.354

σ_ε	μ	ρ	K	r	w	s	$V[a]$
0.4	1	0.9	5.80	0.0369	1.205	0.2464	4.183
0.4	1	0.6	5.63	0.0391	1.192	0.2416	6.983
0.4	1	0.3	5.57	0.0399	1.188	0.2402	10.53
0.4	1	0	5.54	0.0404	1.185	0.2392	15.45

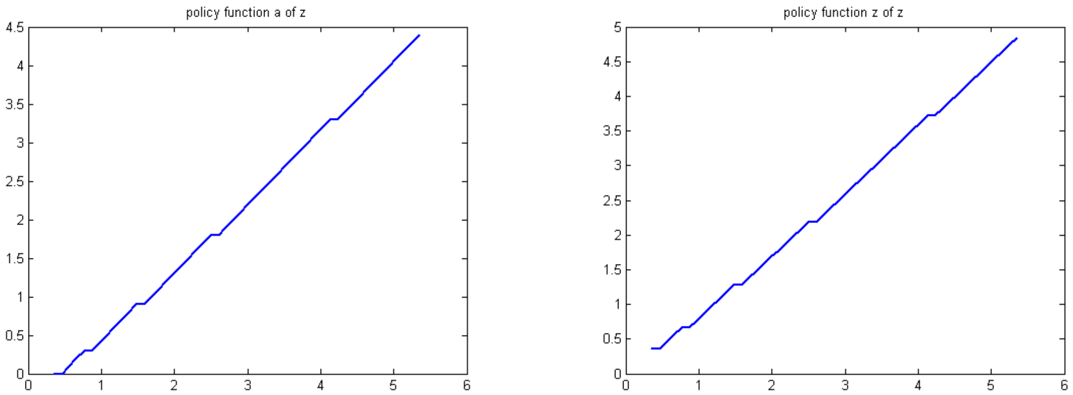
σ_ε	μ	ρ	K	r	w	s	$V[a]$
0.4	2	0.9	6.47	0.0290	1.253	0.2642	3.849
0.4	2	0.6	5.88	0.0359	1.211	0.2486	6.485
0.4	2	0.3	5.72	0.0380	1.199	0.2442	10.27
0.4	2	0	5.65	0.0388	1.194	0.2423	15.30

σ_ε	μ	ρ	K	r	w	s	$V[a]$
0.4	3	0.9	7.40	0.0200	1.315	0.2879	4.063
0.4	3	0.6	6.22	0.0318	1.236	0.2576	6.214
0.4	3	0.3	5.89	0.0357	1.212	0.2490	9.759
0.4	3	0	5.78	0.0371	1.204	0.2460	14.15

Value Functions and Stationary Distribution:



Policy Functions (for lowest labor income):



Market clearing (mu=3,sigmae=0.4):