Roger E. A. Farmer Econ 202

Problem Set 4

1. Consider the following three person pure exchange economy with three commodities. The economy is populated by three people, Mr. A, Mr. B and Mr. C. There are three goods, denoted by subscripts 1,2 and 3. The preferences of Mr. A, Mr. B and Mr. C are given by: $U^A = \log\left(x_1^A\right) + \log\left(x_2^A\right) + \log\left(x_3^A\right)$, $U^B = x_1^B x_2^B x_3^B$, $U^C = -\frac{1}{x_1^C} - \frac{1}{x_2^C} - \frac{1}{x_3^C}$.

The endowments of A,B and C are given by: $\omega^A = \{1,0,0\}$, $\omega^B = \{0,1,0\}$, $\omega^C = \{0,0,1\}$.

Let the price vector be $p = \{p_1, p_2, p_3\}$.

a) Find the set of excess demand functions $f^i(p)$ for i=A,B,C.

b) Find an expression for the aggregate excess demand function $f(p)$.

c) Show that your solution is homogenous of degree zero in p and that it satisfies Walras Law.

2. Find a solution to the social planning problem:

$$\max \frac{U^A}{2} + \frac{\log\left(U^B\right)}{2} + \sqrt{2}\, U^C \quad \text{s.t.} \quad x^A + x^B + x^C \le \omega^A + \omega^B + \omega^C.$$

3. Find values for the transfers τ^A, τ^B, τ^C that cause the competitive equilibrium to implement the social planning problem.

4. Let the following two functions represent the first and second components of the aggregate excess demand function for some three good economy: $f_1 = \frac{p_1 - p_2}{p_3}$, $f_2 = \frac{p_1 p_3}{(p_2)^2}$

Find the third component of the excess demand vector.

5. Consider an overlapping generations economy in which there are two types of agents indexed by superscripts, 1 and 2. The utility functions and endowments of types 1 and 2 are given by:

$$U^1 = \log\left(x_t^{1t}\right) + \beta \log\left(x_{t+1}^{1t}\right), \quad \omega^1 = \{1,0\} \qquad U^2 = \log\left(x_t^{2t}\right) + \beta \log\left(x_{t+1}^{2t}\right), \quad \omega^2 = \{0,1\}$$

where β is a positive parameter. In period 1 there exists an initial generation with the preferences and period 1 endowments:

$$U^1 = -\frac{1}{x_1^{10}}, \quad \omega = 0 \qquad U^2 = \log\left(x_1^{20}\right), \quad \omega = 1,$$

where the superscripts ^{10} and ^{20} mean type 1 generation 0 and type 2 generation 0. Assume that individuals may trade consumption loans with each other. Suppose that a loan of one unit of consumption in period t must be repaid with R_t units of the consumption commodity in period t+1.

i) Find an expression for the intertemporal budget constraint of each type of agent.

ii) Find an expression for the aggregate excess demand function of the young generation.(Hint this is the sum of the excess demands of each type of agent): $\hat{x}_t^t - \omega_t = f(R_t)$.

iii) Define a competitive equilibrium for this economy and find an equilibrium. How many equilibria are there? Are they (is it) efficient? How does you answer depend on the parameter β?

iv) Is there any trade in your equilibrium (equilibria) ?

v) Now suppose that each member of generation 0 is endowed with M units of money. Define a monetary equilibrium for this economy and characterize the set of monetary equilibria as the solution to a difference equation.

vi) What is meant by a stationary equilibrium? How many stationary equilibria are there? How does your answer depend on β?