Problem Set 2

1) Consider a representative agent economy in which the representative family solves the problem:

(1.1) $$\max U = E_1\left\{\sum_{t=1}^{\infty}\beta^{t-1}u\left(C_t, L_t\right)\right\}, \qquad 0<\beta<1,, \qquad \text{where}$$

(1.2) $$u\left(C, L\right) \equiv \frac{\left(C_t^{a}\left(1-L_t\right)^{1-a}\right)^{1-\rho}-1}{1-\rho}, \quad \rho>0, \ 0<a<1 \qquad \text{subject to}$$

(1.3) $$K_{t+1} = K_t\left(1-\delta\right)+Y_t-C_t, \quad t=1,\ldots\infty \qquad K_1=\bar{K},$$

(1.4) $$A_t = A_{t-1}{}^{\lambda}\exp\left(e_t\right), \qquad t=1,\ldots\infty, \qquad 0<\lambda<1, \quad A_0=\overline{A}$$

(1.5) $$Y_t = A_t K_t{}^{\alpha}\left[\left(1+g\right)^t L_t\right]^{1-\alpha} \qquad 0<\alpha<1.$$

where K_t is capital, L_t is labor supply, C_t is consumption, Y_t is output, and α, and β are parameters. A_t is an autocorrelated productivity shock with autocorrelation parameter λ and e is an i.i.d. innovation that has mean zero and small bounded support.

a) Prove that the utility function in (1.2) collapses to the logarithmic case when $\rho=1$. (HINT: use L'Hospital's rule).

b) Write down a pair of stochastic first order conditions for the choice of K_{t+1} and L_t.

c) Define the variables

$$k_t \equiv \frac{K_t}{\left(1+g\right)^t}, \quad y_t \equiv \frac{Y_t}{\left(1+g\right)^t} \quad \text{and} \quad c_t \equiv \frac{C_t}{\left(1+g\right)^t}$$

and write the equations of the model (including the first order conditions) in terms of these transformed variables.

d) Set $\left\{e_t=0\right\}_{t=1}^{\infty}$ and provide an algorithm (indicate how you would write computer code) to find the values for the variables k, L, y and c in a balanced growth path, as functions of the parameters.

e) Find linearized expressions for each of the equations of the model.

f) Chris Sims has written code (Gensys) to solve any model that is in the form

(1.6) $$AX_t = BX_{t-1}+C+\Psi_e e_t+\Psi_w w_t$$

Derive expressions for the matrices A, B, C, Ψ_e and Ψ_w for this example and use Gensys to solve the model. Use the parameter values $\beta=0.97$, $g=0.018$, $\rho=2$, $\alpha=1/3$, a=0.5, $\lambda=0.95$, $\delta=0.1$ and draw $\left\{e_t\right\}$ from a normal distribution with zero mean and standard deviation 0.006.