Economics 202A, Homework #1 Anton Cheremukhin

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Exercise 1

1) $y_t = 1 - 0.5y_{t-1}, \qquad y_0 = 1 \qquad \Rightarrow$

$y_t = (-1/2)^t + \sum_{s=0}^{t-1}(-1/2)^s = \left(-\frac{1}{2}\right)^t + \frac{2}{3}[1 - \left(-\frac{1}{2}\right)^t] \longrightarrow \frac{2}{3}, \qquad \textit{stable}$

2) $y_t = 3 + y_{t-1}, \qquad y_0 = 1 \qquad \Rightarrow \qquad y_t = 1 + 3t \longrightarrow \infty, \qquad \textit{no steady state}$

3) $y_t = 1 - y_{t-1}, \qquad y_0 = 0.5 \qquad \Rightarrow \qquad y_t = 0.5 \longrightarrow 0.5, \qquad \textit{unstable}$

4) $y_t = 2 + 1.2y_{t-1}, \qquad |y_\infty| < \infty \qquad \Rightarrow$

$y_t = y_0(1.2)^t + 2\sum_{s=0}^{t-1}(1.2)^s = (10 + y_0)1.\,2^t - 10 \nrightarrow \infty \Rightarrow y_t = y_0 = -10, \textit{ unstable}$

5) $y_t = 1 + (1/3)y_{t-1}, \qquad y_0 = 0 \qquad \Rightarrow \qquad y_t = \sum_{s=0}^{t-1}(1/3)^s = \frac{3}{2}[1 - \left(\frac{1}{3}\right)^t] \longrightarrow \frac{3}{2}, \qquad \textit{stable}$

6) $y_t = 0.2 + 0.5y_{t-1}, \qquad |y_\infty| < \infty \qquad \Rightarrow$

$y_t = y_0(0.5)^t + 0.2\sum_{s=0}^{t-1}(0.5)^s = y_0 0.5^t - 0.4 \times 0.5^t + 0.4 \longrightarrow 0.4 \qquad \forall y_0, \qquad \textit{stable}$

Exercise 2

1) $A = \begin{bmatrix} 0.5 & 0.2 \\ 0 & 0.2 \end{bmatrix} \Rightarrow Q = \begin{bmatrix} 1 & -2/3 \\ 0 & 1 \end{bmatrix}, Q^{-1} = \begin{bmatrix} 1 & 2/3 \\ 0 & 1 \end{bmatrix}, \Lambda = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$

$u_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow z_t = \Lambda^t Q^{-1} u_o = \begin{bmatrix} \frac{5}{3}(\frac{1}{2})^t \\ (\frac{1}{5})^t \end{bmatrix} \Rightarrow$

$u_t = Qz_t = \begin{bmatrix} 1 & -2/3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{5}{3}(\frac{1}{2})^t \\ (\frac{1}{5})^t \end{bmatrix} = \begin{bmatrix} \frac{5}{3}\left(\frac{1}{2}\right)^t - \frac{2}{3}\left(\frac{1}{5}\right)^t \\ \left(\frac{1}{5}\right)^t \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \textit{stable}$

2) $A = \begin{bmatrix} 0.5 & 0.2 \\ 0.8 & 0.2 \end{bmatrix} \Rightarrow Q = \begin{bmatrix} 1 & -0.347 \\ 0.811 & 1 \end{bmatrix}, Q^{-1} = \begin{bmatrix} 0.676 & 0.234 \\ -0.936 & 0.676 \end{bmatrix}, \Lambda = \begin{bmatrix} 0.777 & 0 \\ 0 & -0.077 \end{bmatrix}$

$u_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow z_t = \Lambda^t Q^{-1} u_o = \begin{bmatrix} 0.91(0.777)^t \\ -0.26(-0.077)^t \end{bmatrix} \Rightarrow$

$u_t = Qz_t = \begin{bmatrix} 0.090\,22\,(-0.077\,)^t + 0.91(0.777\,)^t \\ -0.26\,(-0.077\,)^t + 0.738\,01(0.777)^{\,t} \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \textit{stable}$

3) $A = \begin{bmatrix} 0.5 & 0.2 \\ 0.8 & 0.2 \end{bmatrix} \Rightarrow Q = \begin{bmatrix} 1 & -0.219 \\ 3.06 & 1 \end{bmatrix}, Q^{-1} = \begin{bmatrix} 0.598 & 0.131 \\ -1.83 & 0.598 \end{bmatrix}, \Lambda = \begin{bmatrix} 1.113 & 0 \\ 0 & -0.4132 \end{bmatrix}$

$u_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 1 \\ x \end{bmatrix} \Rightarrow z_t = \Lambda^t Q^{-1} u_o = \begin{bmatrix} 0.729(1.113)^t \\ -1.\,232(-0.413)^t \end{bmatrix} \Rightarrow$

$u_t = Qz_t = \begin{bmatrix} 0.269\,81\,(-0.413\,)^t + 0.729\,(1.\,113)^t \\ -1.\,232\,(-0.413\,)^t + 2.\,230\,7(1.\,113)^t \end{bmatrix} \to \begin{bmatrix} \infty \\ \infty \end{bmatrix}, \qquad \textit{no steady state}$

4) $u_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 1 \\ y \end{bmatrix} \Rightarrow z_t = \Lambda^t Q^{-1} u_o = \begin{bmatrix} (0.131\,y + 0.598)(1.113)^t \\ (0.598\,y - 1.\,83)(-0.413)^t \end{bmatrix} \Rightarrow$

need to assume $0.131\,y + 0.598 = 0 \Rightarrow y = -4.\,564\,9 \Rightarrow z_t = \begin{bmatrix} 0 \\ -4.\,559\,8\,(-0.413\,)^t \end{bmatrix} \Rightarrow$

$u_t = Qz_t = \begin{bmatrix} (-0.413\,)^t \\ -4.\,56\,(-0.413\,)^t \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \textit{unstable}$

Exercise 3

$y_t = \alpha y^e_{t+1} + \beta x_t + u_t \qquad y^e_{t+1} = \lambda y^e_t + (1-\lambda) y_t$

a) $y_t = \alpha(\lambda y^e_t + (1-\lambda)y_t) + \beta x_t + u_t \qquad \Rightarrow \qquad \alpha\lambda y^e_t = (1-\alpha(1-\lambda))y_t - [\beta x_t - u_t]$

$\alpha\lambda y^e_{t+1} = (1-\alpha(1-\lambda))y_{t+1} - [\beta x_{t+1} - u_{t+1}] \qquad \Rightarrow$

$\lambda y_t = (1-\alpha(1-\lambda))y_{t+1} - [\beta x_{t+1} - u_{t+1}] + \beta\lambda x_t + \lambda u_t \qquad \Rightarrow$

$$y_t = \{\lambda[\beta x_{t-1} + u_{t-1}] - [\beta x_t - u_t] - \lambda y_{t-1}\}/(1-\alpha(1-\lambda))$$

b) $v_t, u_t \in [-a, a]$

$$(1-\alpha(1-\lambda))y_t + \beta x_t = -\lambda y_{t-1} + \lambda\beta x_{t-1} + \lambda u_{t-1} + u_t$$

$$x_t = \gamma x_{t-1} + \delta + v_t$$

$$\begin{bmatrix} 1-\alpha(1-\lambda) & \beta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} -\lambda & \lambda\beta \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \delta \end{bmatrix} + \begin{bmatrix} \lambda u_{t-1} + u_t \\ v_t \end{bmatrix}$$

In case shocks are $\begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow \begin{bmatrix} y \\ x \end{bmatrix} \to \begin{bmatrix} \lambda - \alpha(-\lambda+1) + 1 & \beta - \beta\lambda \\ 0 & -\gamma + 1 \end{bmatrix}^{-1} \begin{bmatrix} (\lambda+1)a \\ b+\delta \end{bmatrix} =$

$$\begin{bmatrix} a\frac{\lambda+1}{-\alpha+\lambda+\alpha\lambda+1} + (b+\delta)\frac{\beta-\beta\lambda}{\alpha-\lambda+\gamma-\alpha\lambda-\alpha\gamma+\lambda\gamma+\alpha\lambda\gamma-1} \\ \frac{b+\delta}{1-\gamma} \end{bmatrix}$$

Hence, $x_t \in [\frac{\delta-a}{1-\gamma}, \frac{\delta+a}{1-\gamma}]$, *and*

$y_t \in [\frac{-a(\lambda+1)}{-\alpha+\lambda+\alpha\lambda+1} + \frac{(\delta-a)\beta(1-\lambda)}{\alpha-\lambda+\gamma-\alpha\lambda-\alpha\gamma+\lambda\gamma+\alpha\lambda\gamma-1}, \frac{a(\lambda+1)}{-\alpha+\lambda+\alpha\lambda+1} + \frac{(a+\delta)\beta(1-\lambda)}{\alpha-\lambda+\gamma-\alpha\lambda-\alpha\gamma+\lambda\gamma+\alpha\lambda\gamma-1}]$

c) $y_t = \alpha E_t y_{t+1} + \beta x_t + u_t \qquad y_{t+1} = E_t y_{t+1} + w_{t+1} \qquad x_t = \gamma x_{t-1} + \delta + v_t$

$y_t = \alpha y_{t+1} + \beta x_t + u_t - \alpha w_{t+1} \qquad y_{t+1} = [y_t - \beta x_t - u_t]/\alpha + w_{t+1}$

$y_t = \frac{1}{\alpha}y_{t-1} - \frac{\beta}{\alpha}x_{t-1} + w_t - \frac{1}{\alpha}u_{t-1} \qquad x_t = \gamma x_{t-1} + \delta + v_t$

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha} & -\frac{\beta}{\alpha} \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \delta \end{bmatrix} + \begin{bmatrix} w_t - \frac{1}{\alpha}u_{t-1} \\ v_t \end{bmatrix}$$

Steady state $\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 1-\frac{1}{\alpha} & \frac{\beta}{\alpha} \\ 0 & 1-\gamma \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \delta \end{bmatrix} = \begin{bmatrix} \frac{\beta\delta}{(1-\alpha)(1-\gamma)} \\ \frac{\delta}{1-\gamma} \end{bmatrix}$

$$\begin{bmatrix} \frac{1}{\alpha} & -\frac{\beta}{\alpha} \\ 0 & \gamma \end{bmatrix} = \begin{bmatrix} 1 & \frac{\beta}{1-\gamma\alpha} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\alpha} & 0 \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} 1 & \frac{-\beta}{1-\gamma\alpha} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} y_t - \frac{\beta x_t}{1-\alpha\gamma} \\ x_t \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha} & 0 \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} y_{t-1} - \frac{\beta x_{t-1}}{1-\alpha\gamma} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{-\beta\delta}{1-\alpha\gamma} \\ \delta \end{bmatrix} + \begin{bmatrix} w_t - \frac{1}{\alpha}u_{t-1} - \frac{\beta v_t}{1-\alpha\gamma} \\ v_t \end{bmatrix}$$

Rational Equilibrium: $w_t = \frac{1}{\alpha}u_{t-1} + \frac{\beta v_t}{1-\alpha\gamma}, \qquad y_0 = \beta\frac{\delta\alpha+(1-\alpha)x_0}{(1-\alpha\gamma)(1-\alpha)}$

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha} & -\frac{\beta}{\alpha} \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \delta \end{bmatrix} + \begin{bmatrix} \frac{\beta v_t}{1-\alpha\gamma} \\ v_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{\beta}{1-\gamma\alpha} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\alpha} & 0 \\ 0 & \gamma \end{bmatrix}^t \begin{bmatrix} 1 & \frac{-\beta}{1-\gamma\alpha} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \beta\frac{\delta\alpha+(1-\alpha)x_0}{(1-\alpha\gamma)(1-\alpha)} \\ x_0 \end{bmatrix} + \sum_{s=0}^{t-1} \begin{bmatrix} 1 & \frac{\beta}{1-\gamma\alpha} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\alpha} & 0 \\ 0 & \gamma \end{bmatrix}^s \begin{bmatrix} 1 & \frac{-\beta}{1-\gamma\alpha} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\beta v_{t-s}}{1-\alpha\gamma} \\ \delta + v_{t-s} \end{bmatrix}$$

Exercise 4

$y_t = \alpha E_t y_{t+1} + \delta + \psi y_{t-1} + u_t \qquad y_t = E_{t-1} y_t + w_t$

$$\begin{bmatrix} 1 & -\alpha \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ E_t y_{t+1} \end{bmatrix} = \begin{bmatrix} \psi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ E_{t-1} y_t \end{bmatrix} + \begin{bmatrix} \delta \\ 0 \end{bmatrix} + \begin{bmatrix} u_t \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ w_t \end{bmatrix}$$

$$\begin{bmatrix} y_t \\ E_t y_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\alpha}\psi & \frac{1}{\alpha} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ E_{t-1} y_t \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{\alpha}\delta \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{\alpha}u_t \end{bmatrix} + \begin{bmatrix} w_t \\ \frac{1}{\alpha}w_t \end{bmatrix}$$

a) $\begin{vmatrix} 0-\lambda & 1 \\ -\frac{1}{\alpha}\psi & \frac{1}{\alpha}-\lambda \end{vmatrix} = \frac{1}{\alpha}\left(\psi - \lambda + \alpha\lambda^2\right) = 0 \qquad D = (1-4\psi\alpha)/\alpha^2$

b) $\lambda = \frac{1}{2\alpha}(1 \pm \sqrt{1-4\psi\alpha}) \qquad$ *i.e. roots will be real if* $4\psi\alpha < 1$

c) $\begin{bmatrix} 0 & 1 \\ -\frac{3}{4} & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2/3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 3/2 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & -\frac{1}{2} \\ -\frac{3}{4} & \frac{3}{2} \end{bmatrix}$

d) $\begin{bmatrix} \frac{3}{4}y_t - \frac{1}{2}E_t y_{t+1} \\ -\frac{3}{4}y_t + \frac{3}{2}E_t y_{t+1} \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 3/2 \end{bmatrix} \begin{bmatrix} \frac{3}{4}y_{t-1} - \frac{1}{2}E_{t-1}y_t \\ -\frac{3}{4}y_{t-1} + \frac{3}{2}E_{t-1}y_t \end{bmatrix} + \begin{bmatrix} \delta \\ -3\delta \end{bmatrix} + \begin{bmatrix} u_t \\ -3u_t \end{bmatrix} + \begin{bmatrix} -\frac{1}{4}w_t \\ \frac{9}{4}w_t \end{bmatrix}$

Rational equilibrium: $z_t = \frac{3}{2}z_{t-1} - 3\delta + \frac{9}{4}w_t - 3u_t$

$z_t = -\frac{3}{4}y_t + \frac{3}{2}E_t y_{t+1} = 6\delta \qquad E_t y_{t+1} = 4\delta + \frac{1}{2}y_t$

$a_t = \frac{9}{4}w_t - 3u_t = 0 \qquad w_t = \frac{4}{3}u_t$

$y_t = 4\delta + \frac{4}{3}u_t + \frac{1}{2}y_{t-1} = (\frac{1}{2})^t y_0 + \sum_{s=0}^{t-1}(\frac{1}{2})^s(4\delta + \frac{4}{3}u_{t-s})$

Exercise 5

$p_t = (2/5)x_t + (1/5)E_t p_{t+1} + 1 + u_t$

$x_t = (1/5)p_t + (1/2)x_{t-1} + 1 + v_t, \qquad x_0$

$p_{t-1} = (2/5)x_{t-1} + (1/5)E_{t-1}p_t + 1 + u_{t-1} \qquad p_t = E_{t-1}p_t + w_t$

$$\begin{bmatrix} -1/5 & 0 & 1 \\ 0 & -1/5 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} p_t \\ E_{t-1}p_t \\ x_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/2 \\ -1 & 0 & 2/5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ E_{t-2}p_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} v_t \\ u_{t-1} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w_t \end{bmatrix}$$

$$\begin{bmatrix} p_t \\ E_{t-1}p_t \\ x_t \end{bmatrix} = \begin{bmatrix} 5 & 0 & -2 \\ 5 & 0 & -2 \\ 1 & 0 & \frac{1}{10} \end{bmatrix} \begin{bmatrix} p_{t-1} \\ E_{t-2}p_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} -5 \\ -5 \\ 0 \end{bmatrix} + \begin{bmatrix} -5u_{t-1} \\ -5u_{t-1} \\ v_t - u_{t-1} \end{bmatrix} + \begin{bmatrix} w_t \\ 0 \\ \frac{1}{5}w_t \end{bmatrix}$$

Eigenvectors: $\left\{ \begin{bmatrix} -\frac{1}{\sqrt{1601}-51}\left(-5\sqrt{1601}+205\right) \\ 1 \end{bmatrix} \right\} \leftrightarrow \frac{51}{20} - \frac{1}{20}\sqrt{1601},$

$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \leftrightarrow 0, \qquad \left\{ \begin{bmatrix} \frac{1}{20}\sqrt{1601} + \frac{49}{20} \\ -\frac{1}{\sqrt{1601}+51}\left(-5\sqrt{1601}-205\right) \\ 1 \end{bmatrix} \right\} \leftrightarrow \frac{1}{20}\sqrt{1601} + \frac{51}{20}$

$$\begin{bmatrix} 5 & 0 & -2 \\ 5 & 0 & -2 \\ 1 & 0 & \frac{1}{10} \end{bmatrix} = \begin{bmatrix} 0 & 0.6983 & 0.3793 \\ 1 & 0.6983 & 0.3793 \\ 0 & 0.1569 & 0.8440 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4.5506 & 0 \\ 0 & 0 & 0.5494 \end{bmatrix} \begin{bmatrix} -1.0 & 1 & 0 \\ 1.5928 & 0 & -0.7158 \\ -0.2961 & 0 & 1.3179 \end{bmatrix}$$

$$\begin{bmatrix} -p_t + E_{t-1}p_t \\ 1.5928p_t - 0.7158x_t \\ -0.2961p_t + 1.3179x_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4.5506 & 0 \\ 0 & 0 & 0.5494 \end{bmatrix} \begin{bmatrix} -p_{t-1} + E_{t-2}p_{t-1} \\ 1.5928p_{t-1} - 0.7158x_{t-1} \\ -0.2961p_{t-1} + 1.3179x_{t-1} \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 \\ -7.964 \\ 1.4805 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.7158v_t - 7.2482u_{t-1} \\ 1.3179v_t + 0.1626u_{t-1} \end{bmatrix} + \begin{bmatrix} -w_t \\ 1.4496w_t \\ -0.03252w_t \end{bmatrix}$$

(2) $z_t = 4.5506z_{t-1} - 0.7158v_t + 1.4496w_t - 7.2482u_{t-1} - 7.964$

$z_t = 7.964/3.5506 = 2.243$

$\mathbf{w}_t = (7.2482u_{t-1} + 0.7158v_t)/1.4496 = \mathbf{0.49379v}_t\mathbf{+5u}_{t-1}$

$p_t = (0.7158x_t + 2.243)/1.5928 = 0.44940x_t + 1.4082$

(3) $y_t = 0.5494y_{t-1} + 1.3179v_t - 0.03252w_t + 0.1626u_{t-1} + 1.4805$

$y_t = -0.2961p_t + 1.3179x_t = -0.2961(0.44940x_t + 1.4082) + 1.3179x_t = 1.1848x_t - 0.41697$

$\mathbf{y}_t = 0.5494(1.1848x_{t-1} - 0.41697) + 1.3179v_t - 0.03252(0.49379v_t + 5u_{t-1})$

$+0.1626u_{t-1} + 1.4805 = \mathbf{1.3018v}_t\mathbf{+0.65093x}_{t-1}\mathbf{+1.2514}$

$\mathbf{x}_t = (0.41697 + 1.3018v_t + 0.65093x_{t-1} + 1.2514)/1.1848 = \mathbf{1.0988v}_t\mathbf{+0.5494x}_{t-1}\mathbf{+1.4081}$

$\mathbf{p}_t = 0.44940(1.0988v_t + 0.5494x_{t-1} + 1.4081) + 1.4082 = \mathbf{0.4938v}_t\mathbf{+0.2469x}_{t-1}\mathbf{+2.041}$