Economics 201C
Spring 2006
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Homework 6

1. Price-taking equilibrium and real externalities

There is one consumer with quasilinear utility

$$v(x; z) = 2(x^{1/2} + \alpha z^{1/2}),$$

where $\alpha \geq -1$; and there is one producer with cost $c(y) = y^2/2$. The quantity $z = f(y)$ represents the *added consequences* for utility from producing y units of the output good. The added consequences are negative (positive) when $\alpha < 0$ ($\alpha > 0$). Production of the output good satisfies demand for the consumption good.

Let $v^*(p|\bar{z}) = \max\{v(x'; \bar{z}) - px' : x' \geq 0\}$ be the utility maximizing choice when the price of the consumption good is p and the quantity of z is fixed at $\bar{z}$, which represents the fact that the consumer does not choose the quantity of z. Instead, the consumer regards $\bar{z}$ as a parameter. (Note: $v^*(p|\bar{z})$ is based on NO other sources of wealth such as profit shares.)

(a) Define price-taking equilibrium for the model ignoring the specific functional forms. (The consumer receives the profits of the producer.) What are the restrictions among $x, y,$ and z?

(b) Calculate price-taking equilibrium for the specific functional forms as a function of α when $f(y) = y$.

(c) What is the criterion for efficiency for the model with the functional forms given above? What are the first-order conditions for efficiency for the functions given above when $f(y) = y$.

(d) Under what conditions is too much (too little) output produced and consumed in equilibrium? Give an heuristic explanation in terms of marginal private versus social benefits and costs.

(e) *Pigovian remedy*: Find a tax or subsidy imposed on the consumer depending on the value of α, i.e., the consumer pays $(p + t)$ when the producer receives p, that would bring marginal private and social benefits into equality so that the modified equilibrium in (b) would be efficient. (Taxes collected are returned to the consumer as a lump sum; similarly, subsidies paid are financed by lump sum taxes on the consumer.)

(f) *Complete markets remedy*: There is a market for z; its unit price is r. Hence, $v^*(p, r) = \max\{v(x', z') - px' - rz' : x', z' \geq 0\}$. The producer's profit is $\pi(p, r) = \max\{py' + rf(y') - c(y') : y' \geq 0\}$. When $f(y) = y$, calculate price-taking equilibrium for the explicit functional forms and show that it is efficient.

2. Resource Allocation for a Public Park

A fixed number of acres of land is to be made into a public park with two possible uses, as sports fields and as a botanical gardens. Let $x \in [0, 1]$ be the fraction of the land devoted to sports fields. Assume that no matter what the value of x, costs will be the same and for purposes of simplification assume these costs are *zero*. There are n users of the park, each with a utility function

$$v_i(x) + m_i,$$

where v_i is strictly concave. Not everyone is a sports enthusiast in the sense that not every v_i is increasing in x. For example,

$$v_i = a_i x - b_i x^2,$$

$a_i, b_i \geq 0$, $0 \leq a_i \leq 2b_i$ might be the typical form of a utility function. For the questions below, you regard v_i as a general strictly concave (but not increasing) function. To make explicit computations, let each v_i be of the above quadratic form.

(a) Set up the maximization problem defining an optimal choice of x for $v = (v_1, \ldots, v_n)$ and use it to derive the first-order conditions for a maximum. Assume that the optimal x is in the interior.

(b) Describe a price-taking (Lindahl) equilibrium for this problem where individuals have to "buy" x. (Suggestion: Normalizing the price of m to unity, what are the other prices and the market-clearing conditions?) Show that this price-taking equilibrium will satisfy the first-order conditions of (a).

(c) Define the marginal product mechanism for this class of problems, i.e., for the preferences given by $(v|v_i') = (v_1, \ldots, v_{i-1}, v_i', v_{i+1}, \ldots, v_n)$ how are $x(v|v_i')$ and $m_i(v|v_i')$ determined? After stating the condition implying that individuals have an incentive to reveal their preferences truthfully, show that this condition would be satisfied if individuals received their marginal products.

(d) Both the Lindahl and marginal product allocation schemes agree on the value of $x(v)$ given the *announced* v, but they differ in terms of the value of money payment $m_i(v)$. Write $m_i^L(v)$ as the money payment in the Lindahl mechanism and $m_i^L(v) + \Delta_i(v)$ as the money payment in the marginal product mechanism. Show that $\sum \Delta_i(v) \leq 0$, i.e., the cost of providing individuals with the incentive to reveal truthfully is that they must pay more money.

(e) There are preferences $v^* = (v_1^*, \ldots, v_n^*)$ at which $\Delta_i(v^*) = 0$ for all i, i.e., the Lindahl and marginal product money payments agree. Give an example of such a v^* and demonstrate why $\Delta_i(v^*) = 0$. (Suggestion: Look for a v^* such that $m_i^L(v^*) = 0$.)