Econ201C Homework #4 Anton Cheremukhin

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Exercise 1

a) If both choose #3, then they are effectively in a 4x4 Edgeworth box, with Leontiev preferences, and the starting point is in the corner. Any point on the diagonal is an equilibrium. That means both prices and quantities are not unique: $(z_1, z_2)^A = (z - 4, z)$, $(z_1, z_2)^A = (4 - z, -z)$, $\frac{p_2}{p_1} = \frac{4-z}{z}$, $z \in [0, 4]$. Sum of utilities is 4.

b) If one of the guys switches to #2, the economy for the pair will be described by a 3x4 Edgeworth box, and there will be a two-dimensional set of equilibria. For each equilibrium in (a) we can find a better one in (b) for at least one guy, and vice versa.

c) If both types choose occupation #1, then they have a 2x2 box and they start in the middle of the diagonal. Autarky is efficient and is the only PTE allocation. It is supported by any price vector. If one of the guys chooses to move to a different occupation, he can increase his consumption of one of the goods and decrease consumption of the other. The other guy remains in autarky. Hence, either of them will be worse off by changing occupation.

d) Concluding, we state that the equilibrium occupation is not pareto-optimal.

Exercise 2

a) A play is Pareto-efficient, if none of the agents can be made better off without making the other worse off. This is represented by a Lagrangian, $L = \lambda_1 v_1(z_1, z_2) + \lambda_2 v_2(z_2, z_1)$. The multipliers must be non-negative since we maximize both utilities, and we can normalize them in any way we like, say, by making them sum up to one. Concavity of $v_i()$ is important because, we want the utility-possibility frontiere to be convex. Otherwise the solution to $[\max L]$ will be a corner solution, or we won't get all the points at least. The result does not require quasilinearity.

b) FOC for efficiency: $\lambda_1 dv_1(z_1, z_2) + \lambda_2 dv_2(z_2, z_1) = 0$. This implies, that there is a trade-off of utilities: by increasing one we decrease the other.

c) FOC for Nash equilibrium: $\frac{\partial v_i(z_i, z_{-i})}{\partial z_i} = 0$ for any i.

$v_i(z_i, z_{-i}) = A_{ii} z_i - B_{ii} z_i^2/2 + A_{i-i} z_{-i} - B_{i-i} z_{-i}^2/2$ Nash: $A_{ii} - B_{ii} z_i = 0$ $z_i^e = \frac{A_{ii}}{B_{ii}}$.

PO: $\lambda_1 dz_1 (A_{11} - B_{11} z_1) + \lambda_1 dz_2 (A_{12} - B_{12} z_2) + (1 - \lambda_1) dz_2 (A_{22} - B_{22} z_2) +$

$+ (1 - \lambda_1) dz_1 (A_{21} - B_{21} z_1) = 0$ $\begin{cases} \lambda_1 (A_{11} - B_{11} z_1) + (1 - \lambda_1)(A_{21} - B_{21} z_1) = 0 \\ \lambda_1 (A_{12} - B_{12} z_2) + (1 - \lambda_1)(A_{22} - B_{22} z_2) = 0 \end{cases}$

$z_1^o = \frac{\lambda_1 A_{11} + (1-\lambda_1) A_{21}}{\lambda_1 B_{11} + (1-\lambda_1) B_{21}}$ $z_2^o = \frac{\lambda_1 A_{12} + (1-\lambda_1) A_{22}}{\lambda_1 B_{12} + (1-\lambda_1) B_{22}}$, this is for any weights summing up to one.

For equal weights we get $z_i^o = \frac{A_{ii} + A_{-ii}}{B_{ii} + B_{-ii}}$.

d) $z_1^e \curlyvee z_1^o$ $\Leftrightarrow$ $\frac{A_{11} + A_{21}}{B_{11} + B_{21}} \curlywedge \frac{A_{11}}{B_{11}}$ $\Leftrightarrow$ $\frac{A_{21}}{B_{21}} \curlywedge \frac{A_{11}}{B_{11}}$.

e) There is an externality, which causes the social benefit not to be equal to private benefit. The optimal choice by the two agents of the same quantity is different. The equality case is just an accidental result, reflecting a coincidence of optimality of the same quantity for both agents.

Exercise 3

a) The necessary and sufficient condition for z to be PO is that you cannot do better off to both by moving in any direction: $Dv(z,d) \le 0$ for any d.

b) Nash equilibrium requires that if we change the choice of one of the agents, he is not better off. So by moving along the axis the guy's utility does not increase: $Dv_i(z, d_i) \le 0$, where $d_1 = (\alpha, 0), d_2 = (0, \alpha)$ for any α. Because both functions are concave, it is enough to assume $\alpha = \pm 1$.

c) If $Dv_i(z_0, d_i) = Dv(z_0, d_i) \le 0$ for all i, then the allocation z_0 is a Nash equilibrium, and it cannot be improved by along these directions. That means, that harmfull effects of externalities are eliminated along these directions.

d) However, since the function is not necessarily differentiable, it could be that $Dv(z_0, d_1 + d_2) > Dv(z_0, d_1) + Dv(z_0, d_2)$. That means, it is only a necessary, but not a sufficient condition for efficiency. We could get $Dv(z_0, d_1 + d_2) > 0$.

e) $v_i = z_{-i}(A_i z_i - B_i z_i^2/2) - z_{-i}^{\frac{1}{2}}/2 - z_{-i}/4$ $\quad \frac{\partial v_i}{\partial z} = \left[z_{-i}(A_i - B_i z_i), A_i z_i - B_i z_i^2/2 - z_{-i}^{-\frac{1}{2}}/4 - 1/4\right]$

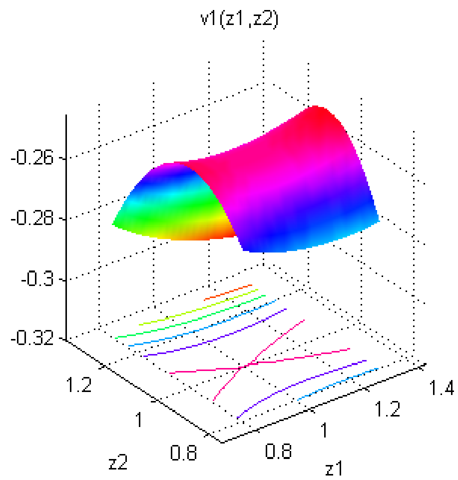
$\frac{\partial^2 v_i}{\partial z \partial z'} = \begin{bmatrix} -z_{-i} B_i & A_i - B_i z_i \\ A_i - B_i z_i & z_{-i}^{-3/2}/8 \end{bmatrix}$. Onw diagonal element is positive, the other - negative.

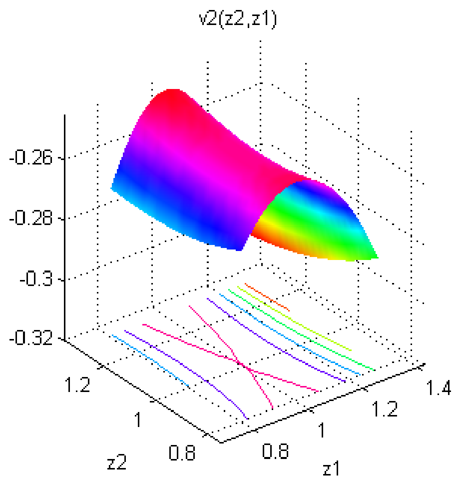
f) $v_i|_{A=1,B=1,z_{-i}=1} = z_i - z_i^2/2 - 3/4 \le -1/4$ is uniquely maximized at $z_i = 1$.

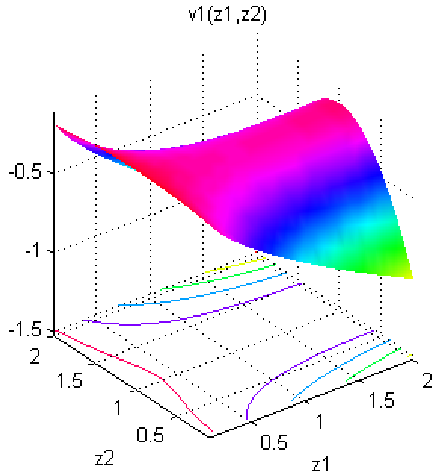
g) $\left.\frac{\partial v_i}{\partial z_i}\right|_{A=1,B=1,z=1} = z_{-i}(1 - z_i)|_{z=1} = 0$ - it is a Nash equilibrium.

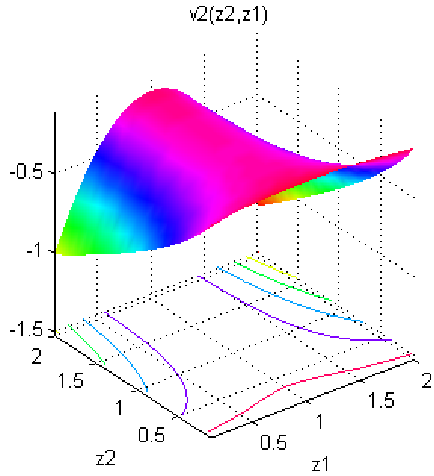
$\left.\frac{\partial v}{\partial z_i}\right|_{A=1,B=1,z=1} = \frac{1}{2} z_{-i}(1 - z_i) + z_{-i} - z_{-i}^2/2 - z_i^{-\frac{1}{2}}/4 - 1/4\Big|_{z=1} = 0$ - satisfies (c).

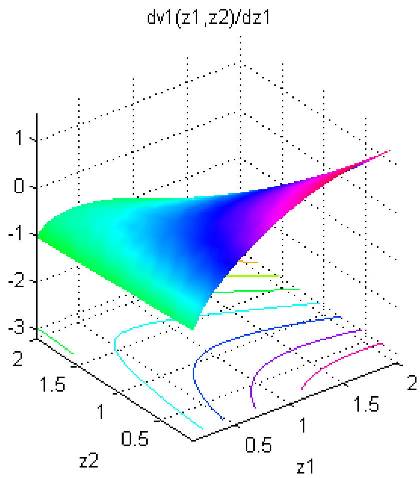
Though it is a maximum along the choice variable for the maximizer, it is a minimum for him along the choice of the other agent. The following graphs illustrate the idea: the local and global behavior of utilities, the derivatives and the pareto-efficient allocation.

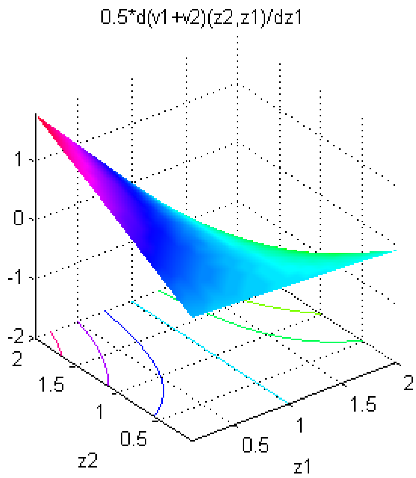


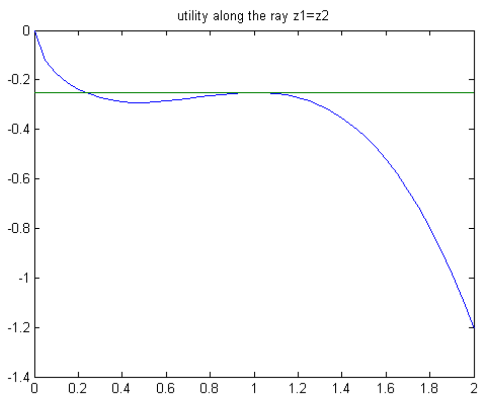












h) There is a Pareto-efficient allocation $z_{-i}\left(z_i - z_i^2/2\right) - z_{-i}^{\frac{1}{2}}/2 - z_{-i}/4\Big|_{z_i=z_{-i}=0} = 0$ - the maximum value over non-negative z.