Econ201C Homework #2 Anton Cheremukhin

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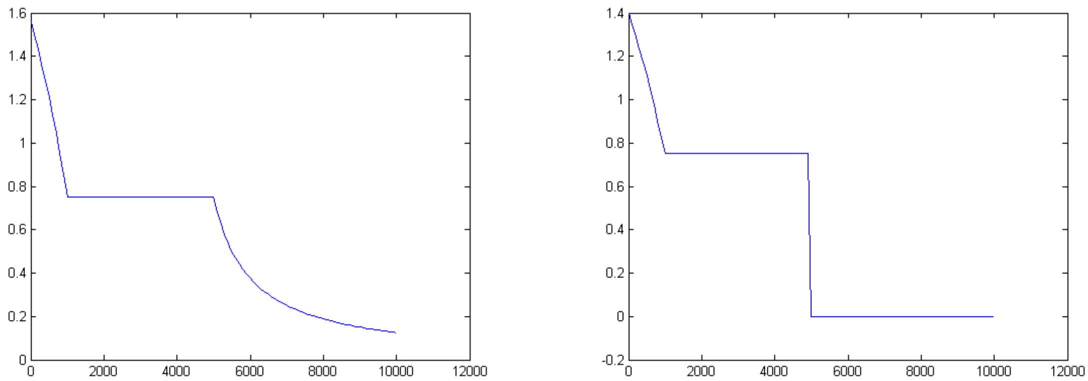
Exercise 1

a) $h(A,b,c) = \max\{cx|\ Ax \le b, x \ge 0\} = \min\{yb|\ yA \ge c, y \ge 0\}$
Primal: $\max\{40x_1 + 70x_2|\ x_1 + x_2 \le 100, 10x_1 + 50x_2 \le 4000, x_i \ge 0\}$
Dual: $\min\{100y_1 + 4000y_2|\ y_1 + 10y_2 \ge 40, y_1 + 50y_2 \ge 70, y_i \ge 0\}$
$x = \begin{bmatrix} 25 & 75 \end{bmatrix}' \qquad y = \begin{bmatrix} 32.5 & 0.75 \end{bmatrix} \qquad h = 6250$
b) $\partial h / \partial b_2 = y_2 = 0.75$ - the gross return in both cases.
c) The range is $[1000, 5000]$. I.e. $t_1 = 3000$, $t_2 = 1000$.
d) Below the range the true value is higher and above the range the true value is lower:



e) $b = [100, 4000]$, $a = [100, 4000 + t]$
$\partial h = \lim \frac{h(a)-h(b)}{t} = \lim \frac{y(a)a - y(b)b}{t} = \lim \frac{y_2(a)(b_2+t) - y_2(b)b_2}{t} = y_2(b)$
$\Delta h = \frac{h(a)-h(b)}{t} = \frac{y(a)a-y(b)b}{t} = \frac{y_2(a)(b_2+t)-y_2(b)b_2}{t} = y_2(b)$, in the range where $y(a) = y(b)$.
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f) For $b_2 = 5000$ we get $x = \begin{bmatrix} 0 & 100 \end{bmatrix}'$, $y = \begin{bmatrix} [32.5; 70] & [0.75; 0] \end{bmatrix}$, $h = 7000$.
Here you can decrease b_2 till 1000 and get a gross rate of 0.75 or increase b_2 and get 0.
This is the switching point: the subdifferential is an interval. Any point in it is a price.
But once we move a little, only the extremes reflect the returns.
For $b_2 = 5001$ we get $x = \begin{bmatrix} 0 & 100 \end{bmatrix}'$, $y = \begin{bmatrix} 70 & 0 \end{bmatrix}$, $h = 7000$.

Exercise 2

a) Primal: $\max\{2x_1 + x_3 | -5x_1 + 2x_2 \le 0, 2x_1 - 3x_2 + 2x_3 \le 0, x_1 + x_2 - 3x_3 \le 0, 0 \le x_i \le 1\}$
Dual:
$\min\{y_4 + y_5 + y_6 | -5y_1 + 2y_2 + y_3 + y_4 \ge 2, 2y_1 - 3y_2 + y_3 + y_5 \ge 0, 2y_2 - 3y_3 + y_6 \ge 1, y_i \ge 0\}$
b) $x = \begin{bmatrix} 0.87 & 1 & 0.625 \end{bmatrix}'$ $\quad y = \begin{bmatrix} 0 & 0.875 & 0.25 & 0 & 2.375 & 0 \end{bmatrix}$ $\quad h = 2.375.$
c) Equilibrium is a set of allocations x and prices y
s.t. 1) x solves firms' problems given y and 2) markets clear.
Firm problems:
$\pi_1 = \max_{x_1}\{x_1(2 - (-5y_1 + 2y_2 + y_3)) \quad | 0 \le x_1 \le 1\}$
$\pi_2 = \max_{x_2}\{x_2(0 - (2y_1 - 3y_2 + y_3)) \quad | 0 \le x_2 \le 1\}$
$\pi_3 = \max_{x_3}\{x_3(1 - (2y_2 - 3y_3)) \quad | 0 \le x_3 \le 1\}$
Markets clear if: $-5x_1 + 2x_2 = 0, \quad 2x_1 - 3x_2 + 2x_3 = 0, \quad x_1 + x_2 - 3x_3 = 0$
d) The allocations $x = \begin{bmatrix} 0.87 & 1 & 0.625 \end{bmatrix}$ and prices $y = \begin{bmatrix} 0 & 0.875 & 0.25 \end{bmatrix}$ constitute an equilibrium. The equilibrium profits are $\pi = \begin{bmatrix} 0 & 2.375 & 0 \end{bmatrix}$. That's because either π_i are zero, because the firm has an interior solution (the capacity constraint is not binding), or $x_i = 1$ and then π_i is equal y_{i+3} by construction.

Exercise 3

$u(x,m) = v(x) + m \to \max$ s.t. $px + m = w$
$L = v(x) + m + \lambda(w - m - px) \qquad$ FOC: $v'(x) = p,\ \lambda = 1$
c) Demand $x(p,w) = v'^{-1}(p)$ is independent of income.
d) By Slutsky $\frac{\partial x(p,w)}{\partial p} = \frac{\partial x(p,u)}{\partial p} - x(p,w)\frac{\partial x(p,w)}{\partial w}$. The income effect is equal to zero here. The substitution effect is always negative: $\frac{\partial x(p,u)}{\partial p} \le 0$. Hence, price elasticities: $\epsilon = \frac{\partial x(p,w)}{\partial p}\frac{p}{x(p,w)} \le 0$.
a) Indirect utility: $\quad u(p,w) = v(x) + w - px = v(v'^{-1}(p)) - pv'^{-1}(p) + w = \alpha(p) + w$
b) Expenditure function: $\quad e(p,u) = px + m = u - \alpha(p) = \beta(p) + u$

Exercise 4

$D(p) = \arg\max\{v(x) + m | px + m = 0\}$
a) By definition of indirect utility $v^*(p) = \sup[v(x) + m | px + m = 0] = v(z) - pz$ which given continuity of $v(\cdot)$ implies that $(z,m) \in D(p)$ because max = sup and it is the same optimization problem.
By definition of subdifferential $p \in \partial v(z) = \{p : v(z) + p(y - z) \ge v(y), \forall y\}$ can be rewritten as $v(z) - pz \ge v(y) - py, \forall y$ which immediately implies $v^*(p) = v(z) - pz$.
b) $v \equiv -c$. The objective is to choose $(z, c(z)) \in Y$ maximizing profits given prices $(p, 1)$. Now let outputs be negative ($z < 0$) and inputs positive ($c(z) > 0$): -$(p,1)(z, c(z)) = -pz - c(z) = v(z) - pz$ - revenue from non-money trade z minus the money input cost of trade constitutes profits. Hence, $(z, -v(z))$ is profit maximizing iff $v(z) - pz = \sup[v(x) + m | px + m = 0] = v^*(p)$ - the profit function.