Econ201B Homework #1 Anton Cheremukhin

January 22, 2006

Exercise 1 *Static Games*

Horizontal player strategies are denoted as a,b,c, Vertical - as A,B,C.
(strict or weak; iterated strict; Nash; trembling hand perfect)
a) none; none; aA,bB,$\frac{2a+b}{3}\frac{A+2B}{3}$; all thp
b) B>A,b>a; same; bB; thp
c) none; a>b,A>B,a>c,A>C; aA; thp
d) Aa,Bb weak; same; aA,bB,$\frac{qa+(1-q)b}{1}\frac{A+B}{2}q \in [0,1]$; all thp except $q=0$ and $q=1$.

Exercise 2 *Dynamic Games*

The nature could be moved to termanal nodes and then mathematical expectation could be taken among the alternatives. Assigning strategies by a - left, A - 5 5, C - 7 3, c - 1 2, E - 2 2, e - ∀ 4. This simplification leads to the following normal form:

2\1	ace	acf	ade	adf	bce	bcf	bde	bdf
ACE	5 5	5 5	5 5	5 5	7 3	7 3	7 3	7 3
ACF	5 5	5 5	5 5	5 5	7 3	7 3	7 3	7 3
ADE	5 5	5 5	5 5	5 5	1 2	1 2	2 2	2 2
ADF	5 5	5 5	5 5	5 5	1 2	1 2	$\frac{3}{2}$ 4	$\frac{1}{2}$ 1
BCE	0 0	0 0	0 0	0 0	7 3	7 3	7 3	7 3
BCF	0 0	0 0	0 0	0 0	7 3	7 3	7 3	7 3
BDE	0 0	0 0	0 0	0 0	1 2	1 2	2 2	2 2
BDF	0 0	0 0	0 0	0 0	1 2	1 2	$\frac{3}{2}$ 4	$\frac{1}{2}$ 2

We could remove BDE due to strict dominance and further simplify to behavioral strategies:

2\1	a∀∀	bc∀	bdc	bdf
AC∀	5 5	7 3	7 3	7 3
ADE	5 5	1 2	2 2	2 2
ADF	5 5	1 2	$\frac{3}{2}$ 4	$\frac{1}{2}$ 1
BC∀	0 0	7 3	7 3	7 3
BDF	0 0	1 2	$\frac{3}{2}$ 4	$\frac{1}{2}$ 1

Here we can see the following 21 pure NE:
$a\forall\forall AD\forall$, $bc\forall\forall C\forall$, $bdf\forall C\forall$, $bdeBDF$.
Assigning probabilities p,q,r to columns and α,β,γ,δ to rows one can find that the set of mixed NE is: $\left(\frac{bc\forall+bdc}{2},\ \alpha AC\forall + \gamma ADF + \frac{\alpha+12\gamma-1}{11}BC\forall + \frac{10-12\alpha-23\gamma}{11}BDF\right)$,
where $0 \leq \alpha \leq \frac{97}{121}$ and $\frac{1-\alpha}{12} \leq \gamma \leq \frac{10-12\alpha}{23}$.
(5 5) is the only subgame perfect equilibrium.

Exercise 3 *Dominance and Nash Equilibrium*

Take some equilibrium which an NE in dominating strategies. Than by adding strategies which are dominated by the equilibrium profile we can't create incentives to divert from it and hence it is a NE in the big game. If a NE profile in the big game uses a strategy which is strictly dominated by some other strategy, than it can't be NE by definition. An example of NE which is lost when removing weakly dominated strategies is the following:

1,1	0,1
1,0	0,0

Here all the four profiles as well as any mixed strategy are NE. However, any pair of strategies could be removed by weak dominance, which leads to loss of all equilibria except one.

Exercise 4 *Backward Induction*

We assume (though not obvious) that the proposal is accepted if votes are equal.

In this case assume there are 2 players. Then the first one will take everything to himself: 100,0. If there are 3 players the first should convince the last player to accept the proposal say by 1 coin. Then the 99,0,1 rule is accepted by 2 out of three players. By induction it is easy to see that for 5 players the optimal proposal is 98,0,1,0,1.

Exercise 5 *Correlated equilibrium*

Consider vertical player. If he is receives an advice to take B than he knows for sure that the other guy is playing a. Then B is the best response.

If, on the other hand, he receives and advice to take A than he the player is playing a or b with equal probabilities. In this case playing A gives him $\frac{0+2}{2} = 1$ on average and B gives $\frac{0+1}{2} = \frac{1}{2}$. Hence, A is better.

Overall that means that he is going to accept the advice. By symmetry the same is true for the horisontal player. I.e. it is a correlated equilibrium.