Econ 201A Microeconomic Theory

Homework 4

1. The Game

Each individual has VNM utility function $v(c) = \ln c$. Bruin fans believe that UCLA will win with probability 0.8. Trojan fans believe that USC will win with probability 0.6. Total Bruin fan wealth is W. Total Trojan fan wealth is 3W.

(a) Let state 1 be a Bruin victory and state 2 a Trojan victory. Let p_s be the price of a claim to \$1 in state *s*. Write down expressions for the total Bruin demand for state 1 claims and the total Trojan demand for state 1 claims. What is the total supply?

(b) Solve for the Walrasian Equilibrium state claims price ratio p_1 / p_2.

(c) Suppose that CalTech fans are also interested in the game. Each of them believes that the two teams are equally likely to win. If total CalTech wealth is $\frac{1}{2}W$ what will the equilibrium price ratio be?

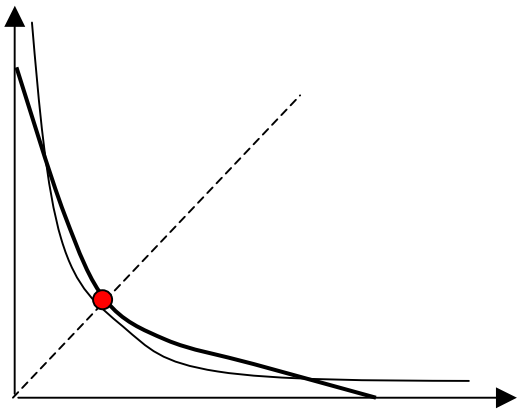
(d) Suppose instead that Caltech wealth is $4W$. How will this affect equilibrium prices?

2. Small and Large gambles

Alex exhibits constant relative risk aversion of degree $\bar{R}$. Alex exhibits constant absolute risk aversion of degree $\bar{A}$. Both have the same riskless wealth.

(a) Explain why Alex has greater absolute risk aversion than Bev if his wealth exceeds $\bar{R} / \bar{A}$.

(b) Henceforth assume that this inequality holds. There are two states, the probability of state 1 is π. Show that the indifference curves through (w, w) must be as depicted below. That is they touch on the 45° line and intersect twice.



(c) Hence comment on which individual is more willing to take on small risks and which is more willing to take on large risks.

3. Asset and state claims prices

There are 2 states and 2 assets. The asset A vector of returns in the two states is $z^A = (\alpha, \beta)$. The asset B return vector is $z^B = (1-\alpha, 2-\beta)$. The two states are equally likely. Each individual has the same VNM utility function $v(c) = \frac{c^{1-R}}{1-R}$, $R > 1$.

(a) Solve for the WE state claims prices.

(b) Solve for the WE asset prices.

(c) Under what conditions will the price of asset A rise relative to the price of asset B as the degree of relative risk aversion increases?

(d) Give the intuition behind this conclusion.

(e) If there are no state clams markets but individuals can trade assets, what will be the equilibrium asset prices? Explain.

4. Asset and state claims prices with more states than assets

Each individual has the same VNM utility function $v(c) = c^{1/2}$. There are 4 equally likely states. The aggregate endowment of state claims is $\omega = (1, 4, 9, 16)$.

(a) Solve for the equilibrium state claims price vector and show that every agent will consume a fraction of the aggregate endowment.

(b) The aggregate endowment is in the form of two assets Asset A has return $z^A = (1,1,1,1)$ while asset B has return $z^B = (0, 3, 8, 15)$. What is the WE asset price ratio?

(c) Suppose that individuals can only trade in asset prices. Is the asset price ration of part (b) still the equilibrium price ratio? If so, explain. If not, why not?