 15 November 2005

Econ 201A

Homework 3, Fall 2005

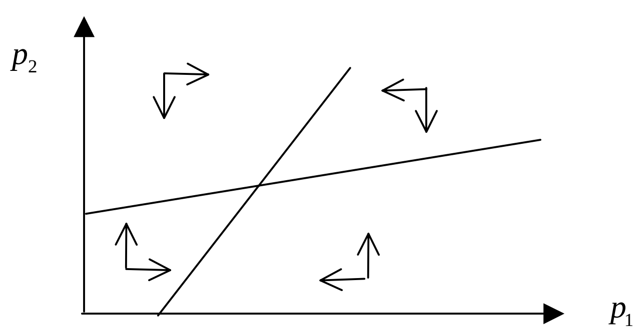
1. Price Dynamics in a 3 consumer 3 commodity Cobb-Douglas model

(a) Alex, Bev and Chuck have Symmetric Cobb-Douglas utility functions. Endowments are as follows. $\omega^A = (1,0,0)$, $\omega^B = (0,1,0)$, $\omega^C = (0,0,1)$. Solve for the demand functions and hence show that if $p_3 = 1$, the excess demand functions for commodity 1 and 2 are as follows.

$$e_1(p) = \frac{1-2p_1+p_2}{3p_1},\ e_2(p) = \frac{1+p_1-2p_2}{3p_2}.$$

(b) Draw the lines $e_1(p) = 0$ and $e_2(p) = 0$ in a diagram with p_1 on the horizontal and p_2 on the vertical axis. What is the sign of $e_1(p)$ and $e_2(p)$ in each "phase" bounded by the two lines? What are the Walrasian Equilibrium prices?

(c) Suppose that the price of commodity *j* rises if there is excess demand and falls if there is excess supply. Confirm that the arrows indicating the direction of change in the two prices must be as depicted below.



Draw a conclusion as to the stability of the price adjustment process.

2. Price Dynamics with no substitution effects

Suppose that modify the model examined in question 1 as follows.

Alex has utility function $U^A = Min\{60x_2^A, x_3^A\}$ and endowment $\omega^A = (0,0,800)$.

Bev has utility function $U^B = Min\{40x_1^B, 3x_3^B\}$ and endowment $\omega^B = (20,0,0)$.

Chuck has utility function $U^C = \{6x_1^C, x_2^C\}$ and endowment $\omega^C = (0,40,0)$.

Again assume that consumers use commodity 3 as the "numeraire" good so $p_3 = 1$.

(d) Show that $x_1^B = \dfrac{60p_1}{40+3p_1}$ and $x_1^C = \dfrac{40p_2}{p_1+6p_2}$, hence the excess demand for commodity 1 is $e_1(p) = \dfrac{60p_1}{40+3p_1} + \dfrac{40p_2}{p_1+6p_2} - 20$.

(e) Show also that the excess demand for commodity 2 is

$$e_2(p) = \frac{800}{60+p_2} + \frac{240p_2}{p_1+6p_2} - 40.$$

(f) Confirm that $p = (40,20)$ clears the markets for commodities 1 and 2. Why does this imply that the market for commodity 3 must also clear?

(g) Show that the linear approximation of the excess demand for commodity 1 at $p = (40,20)$ is

$$\overline{e}_1(p) = \tfrac{1}{16}(p_1 - 40) + \tfrac{1}{16}(p_2 - 20)$$

(h) Solve also for the linear approximation of $e_2(p)$ at $p = (40,20)$.

(i) Draw the lines $\overline{e}_1(p) = 0$ and $\overline{e}_2(p) = 0$ in a diagram with p_1 on the horizontal and p_2 on the vertical axis. Indicate the sign of $\overline{e}_1(p)$ and $\overline{e}_2(p)$ in each "phase" bounded by the two lines.

(j) Using the simple price adjustment process described above, examine the "stability" of the Walrasian Equilibrium (starting at a point where there is excess demand for both commodity 1 and commodity 2.)

Note: This question was adapted from a Working Paper by Charlie Plott et al "Divergence, Closed Cycles and Convergence is Scarf Environments" Cal Tech 2004.

3. Equilibrium Puzzles

Alex has utility function $U^A(x^A) = 3\ln(x_1^A - 4) + \ln(x_2^A - 4)$ and an endowment $\overline{x}^A$. Bev has a utility function $U^B(x^B) = \ln(x_1^B - 4) + 3\ln(x_2^B - 4)$ and an endowment $\overline{x}^B$. For each the consumption set is $X = \{x \mid x \geq (4,4),\ \ x \neq (4,4)\}$

(a) Solve for each consumer's demand function for commodity 1.

HINT: You may wish to define $z_j^h = x_j^h - 4$ and $\overline{z}_j^h = \overline{x}_j^h - 4$, then convert the problem to one where both individuals have standard logarithmic preferences.

(b) Hence show that total demand for commodity 1 is

$$x_1(p) = 8 + \tfrac{3}{4}(\overline{x}_1^A - 4) + \tfrac{1}{4}(\overline{x}_1^B - 4) + \frac{p_2}{p_1}[\tfrac{3}{4}(\overline{x}_2^A - 4) + \tfrac{1}{4}(\overline{x}_2^B - 4)].$$

(b) If $\overline{x}^A = (10,2)$ and $x^B = (2,10)$ show that there is more than one Walrasian Equilibrium price ratio.

(c) What is the set of WE price ratios? Depict these in an Edgeworth Box diagram.

HINT: Both demands must lie in each individual's consumption set.

(d) If $\overline{x}^A = (11,1)$ and $x^B = (1,11)$ show that the unique WE price ratio is 1. Show also that if Alex's endowment of commodity 1 is increased by reducing Bev's endowment, the WE price ratio will decline.

(e) Confirm that Alex's WE consumption of both commodities will decline as his endowment of commodity 1 increases.

(f) If you were Alex's advisor would you encourage him to make a gift of commodity 1 to Bev?

4. Constant Returns to Scale Economy

The production function for commodity *A* is $q_A = (\frac{z_1^A z_2^A}{3})^{1/2}$. The production function for commodity *B* is $q_B = \frac{2}{3} Min\{z_{B1}, z_{B2}\}$. The aggregate endowment of inputs is (300,200).

(a) Solve for the unit cost functions $c_A(r)$ and $c_B(r)$.

Henceforth assume that the output prices are both 1.

(b) Show that $r = (\frac{1}{6}, \frac{1}{2})$ is an equilibrium input price vector and that the unit input demands are $v_A = (3,1)$ and $v_B = (\frac{3}{2}, \frac{3}{2})$.

(c) Solve for the equilibrium outputs.

(d) Depict the unit isoquants in a neat figure and also draw the unit cost line. Why must the unit cost line touch the unit isoquants?

(e) Looking at the figure, can you see why there must be a second pair of input prices that equate unit costs and output prices.

(f) Suppose the aggregate supply on input 2 rises from 200 to 250. Explain why input prices will not change.

(g)* What if the aggregate supply of input 2 rises to 400?