Microeconomic Theory Fall 2005

Homework 2

1. Homothetic Preferences and Equilibrium Prices

(a) Consumer h, $h = 1, ..., H$ has a utility function

$$U(x^h) = (x_1^h)^{1/2}(x_2^h)^{1/2} + \delta(x_3^h)^{2/3}(x_4^h)^{1/3}.$$

Explain why aggregate demand is the same as the demand of a single agent holding the entire endowment. If this endowment is $(100, 400, 200, 200)$ solve for equilibrium prices. (Normalize so that $p_1 = 1$.) What if the endowment is of the form $\omega = (a, 4a, b, b)$ where both parameters are strictly positive?

(b) Consumer h, $h = 1, ..., H$ lives for two periods. There are two commodities in each period. His utility function is

$$U(x^h) = (x_{11}^h)^{1/2}(x_{12}^h)^{1/2} + \delta(x_{21}^h)^{2/3}(x_{22}^h)^{1/3}$$

where x_{tj} is the period *t* consumption of commodity *j*. The aggregate period 1 endowment is $(\omega_{11}, \omega_{12}) = (100, 400)$. The period 2 endowment is $(\omega_{21}, \omega_{22}) = (600, 600)$. The discount factor $\delta = 1/2$ Commodities are non-storable. Solve for equilibrium spot and futures prices if the spot price of commodity 1 is 1.

(c) There are no futures markets but banks lend and borrow at the rate *r*. If the spot price and future spot price of commodity 1 are both 1, what is the equilibrium interest rate?

(d) Suppose that goods are storable. For what values of the "discount factor" δ would there be no profit from storing at these prices, that is, the prices remain equilibrium prices?

2. Competitive and monopoly supply of oil

There are $\bar{q}$ barrels of oil that can be extracted at no cost and consumed either in period 1 or period 2. Demand price functions are $p_1(q_1)$ and $p_2(q_2)$.

(a) If firms are price takers and the interest rate is *r*, explain why equilibrium prices must satisfy $p_1(q_1) = \dfrac{p_2(q_2)}{1+r}$.

(b) If $p_1(q_1) = \frac{b_1}{q_1^{1/e}}$ and $p_2(q_2) = \frac{b_2}{q_2^{1/e}}$, solve for the equilibrium outputs in each period.

(c) Suppose oil is produced by a quantity (and hence) price-setting monopolist whose objective is to maximize the present value of the profit stream. Solve for the monopoly outputs and prices in each period.

(d) Is this a surprising result?

3. Quality and quantity choice

A monopolist can sell q units at a price $p(\theta, q) = a - \frac{q}{\theta}$. The parameter θ is a quality parameter. The higher the quality the higher is the cost of production. Suppose $C(\theta, q) = \theta q$.

(a) Is the profit function $\Pi(\theta, q)$ concave?

(b) Fix θ and solve for the profit maximizing quantity for a given θ.

(c) Solve for the optimal quality and quantity.

(d) Next suppose that the demand price function is $p(\beta, q) = a\beta - q$ and the cost function is $C(\beta, q) = \beta^2 q^2$ where $\beta \in [0, \overline{\beta}]$, where $\overline{\beta}$ is large. This time it is the parameter β that represents quality. Again solve for the profit maximizing quality and quantity.

4. Electricity Pricing

An unregulated utility faces the following demand price functions

Late Night: $p_1 = 605 - 2q_1^2$, Day: $p_2 = 80 - 2q_2$, Late afternoon/evening: $p_3 = 70 - 2q_3$.

The cost per day, per unit of turbine capacity is $c_0 = 20$. The cost of operating the turbines to produce electricity in any given period is 5 per unit so total cost is $C(q) = 5\sum_{j=1}^{3} q_j + 20q_0$.

(a) Write down an expression for the firm's profit and constraints that must be satisfied.

(b) If the monopolist's goal is to maximize profit solve for the profit maximizing outputs and prices.

(c) A regulator commands the monopolist to maximize social surplus. Write down expressions for the area under each demand curve and then solve for the outputs and prices that maximize the social surplus.