Econ 201A Microeconomic Theory

Problem Set 1

1. Cobb-Douglas Models

(a) Bev has a Cobb-Douglas utility function $U(x) = x_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$, where $\alpha_1, \alpha_2, \alpha_3 > 0$. She has an income of I and faces a price vector p. (Unless told otherwise you should always assume prices are strictly positive.) Solve for her demand functions and hence her maximized utility.

(b) A firm has a production function $F(z) = z_1^{\alpha_1} z_2^{\alpha_2} z_3^{\alpha_3}$. The input price vector is r. What is the "cost function" of the firm, that is, the minimized cost of producing q units of output?

2. Indivisible commodity (part (c) changed)

(a) Alex has a utility function $U(x_1, x_2) = (a + x_1)^{\alpha_1} x_2^{\alpha_2}$, $a > 0$. The price vector is p and Alex has an income of I. Solve for his demand functions and hence his maximized utility.

John has a utility function $U(x_0, x_1, x_2) = (a + x_0 + x_1)^{\alpha_1} x_2^{\alpha_2}$, $a > 0$. Commodity zero is indivisible and only one unit can be purchased. That is $x_0 \in \{0,1\}$. The price of one unit is p_0.

(b) Assuming income is high, solve for John's maximized utility and hence indicate the range of prices for which John will purchase the indivisible commodity.

(c) Solve for John's demand functions for all prices and incomes if the parameter $a = 0$.

3. Consumer demand

(a) A consumer has a utility function $U(x_1, x_2) = (\frac{1}{x_1} + \frac{1}{x_2})^{-1}$. Set up the Lagrangian then show that $\frac{p_1 x_1}{p_1^{\frac{1}{2}}} = \frac{p_2 x_2}{p_2^{\frac{1}{2}}} = \frac{1}{\lambda^{\frac{1}{2}}}$

(b) Substitute your answers into the budget constraint and solve $\lambda^{\frac{1}{2}}$. Hence, or otherwise, solve for the demand functions. Finally show that maximized utility is

$$V(p,I)=\frac{I}{(p_1^{1/2}+p_2^{1/2})^2}$$

(c) A consumer has a utility function $U(x)=(\frac{1}{x_1x_3}+\frac{1}{x_2x_3})^{-1}=x_3(\frac{1}{x_1}+\frac{1}{x_2})^{-1}$. Solve for his demand functions.

Hint: Break down the problem into two stages. In stage 1 fix x_3 and consider how to optimally allocate y spent on the first two commodities then appeal to part (b). In stage 2 consider the indirect utility function $U^*(y,x_3)$ and budget constraint $y+p_3x_3=I$.

4. Pareto Efficient Allocations

Consumer h, $h=A,B$ has a utility function $U^h(x_1^h,x_2^h)=(a_1^h+x_1^h)^{\alpha_1}(a_2^h+x_1^h)^{\alpha_2}$.

The aggregate endowment vector is ω.

(a) The allocation $\bar{x}^A,\bar{x}^B$ is Pareto Efficient if $(\bar{x}^A,\bar{x}^B)$ is the solution to the following problem.

$$\underset{x^A,x^B}{Max}\{U^A(x^A)\,|\,x^A+x^B\le\omega,\ U^B(x^B)\ge U^B(\bar{x}^B)\}$$

Explain why this must be true by drawing an Edgeworth Box Diagram..

(b) Suppose $a^A=a^B=0$. Show that the Pareto Efficient allocations in the interior of the Edgeworth Box lie on the diagonal.

(c) For all vectors $a^A,a^B\ge 0$ show that the PE allocations in the interior of the box lie on a line.

(d) Define $y^h=a^h+x^h$, $h=A,B$. Then $U^h=(y_1^h)^{\alpha_1}(y_2^h)^{\alpha_2}$.

Note that $y^A+y^B\le a^A+a^B+\omega$. Then draw a big Edgeworth Box and depict the PE allocations in this diagram. Depict a smaller Edgeworth Box showing the feasible y allocations inside the big one if $a^A=a^B=(2,1)$

(e) Are there any values of a^A,a^B for which there are no PE allocations in the interior of the Edgeworth Box? Explain.