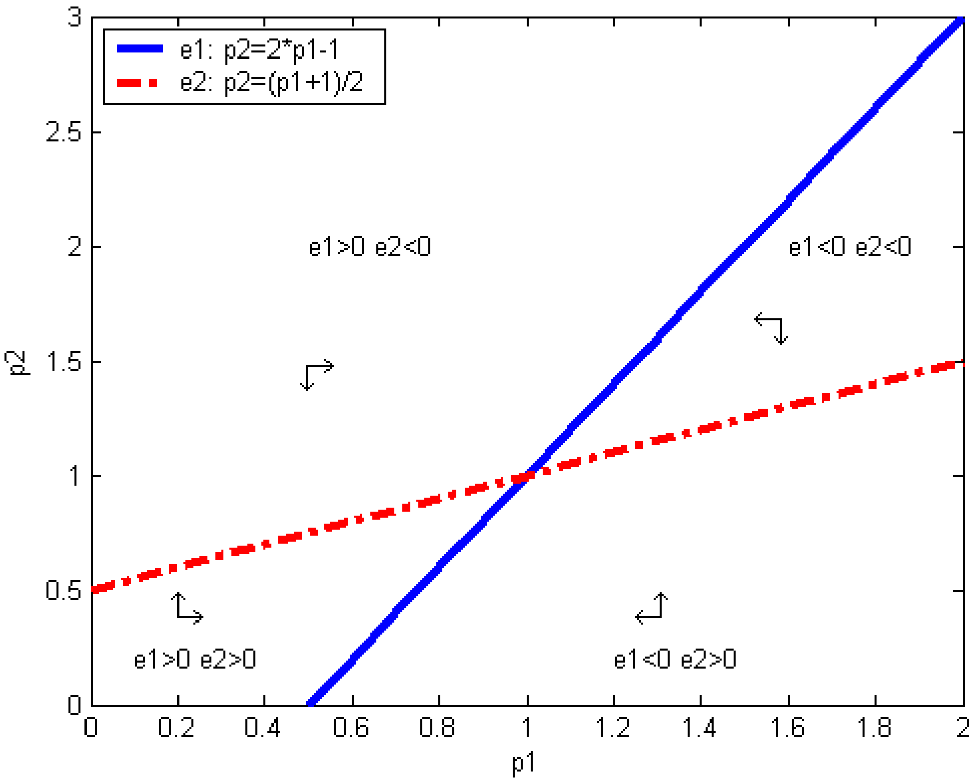
November 28, 2005

Exercise 1

(a) $U^A(x) = U^B(x) = U^C(x) = \ln x_1 + \ln x_2 + \ln x_3$
Endowments: $w^A = (1,0,0)$, $w^B = (0,1,0)$, $w^C = (0,0,1)$.
Using the FOCs: $\frac{\partial U(x)/\partial x_i}{\partial U(x)/\partial x_j} = \frac{p_i}{p_j}$
Get demands: $x_i^j = \frac{I_j}{3p_i} \Rightarrow x_i^A = \frac{p_1}{3p_i}$, $x_i^B = \frac{p_2}{3p_i}$, $x_i^C = \frac{p_3}{3p_i}$
So the excess demands are: $e_i(p) = \frac{p_1+p_2+p_3}{3p_i} - 1$
Hence, assuming $p_3 = 1$ we get: $e_1(p) = \frac{1+p_2-2p_1}{3p_1}$, $e_2(p) = \frac{1+p_1-2p_2}{3p_2}$

(b) $e_1(p) = 0 \Rightarrow 1 + p_2 = 2p_1$, $\quad e_2(p) = 0 \Rightarrow 1 + p_1 = 2p_2$
Walrasian Equilibrium: $p_1 = p_2 = 1$

(c) $e_i > 0 \Rightarrow p_i \uparrow \qquad e_i < 0 \Rightarrow p_i \downarrow$
That means (according to the picture) that the Walrasian Equilibrium is stable and any initial prices converge to it.

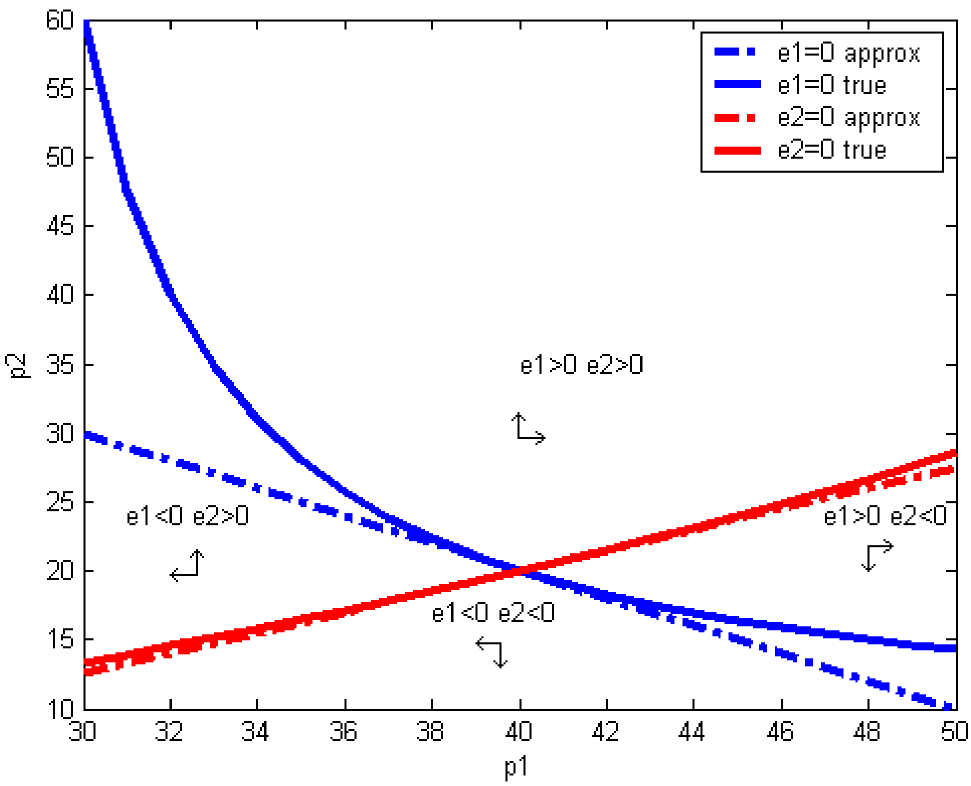


Exercise 2

(d,e) $U^A(x) = \min[60x_2, x_3]$ $\quad w^A = (0, 0, 800)$ $\quad U^B(x) = \min[40x_1, 3x_3]$ $\quad w^B = (20, 0, 0)$
$U^C(x) = \min[6x_1, x_2]$ $\quad w^C = (0, 40, 0)$

They'll always divide their income to buy constant proportions of goods:
$60x_2 = x_3, \qquad p_2x_2 + p_3 60x_2 = 800p_3 \qquad \Rightarrow \qquad x_2^A = \frac{800p_3}{p_2+60p_3}, \qquad x_3^A = \frac{48000p_3}{p_2+60p_3}$
$40x_1 = 3x_3, \qquad p_1x_1 + p_3\frac{40}{3}x_1 = 20p_1 \qquad \Rightarrow \qquad x_1^B = \frac{60p_1}{3p_1+40p_3}, \qquad x_3^B = \frac{800p_1}{3p_1+40p_3}$
$6x_1 = x_2, \qquad p_1x_1 + p_2 6x_1 = 40p_2 \qquad \Rightarrow \qquad x_1^C = \frac{40p_2}{p_1+6p_2}, \qquad x_2^C = \frac{240p_2}{p_1+6p_2}$
Assuming $p_3 = 1$ the excess demands are: $e_1(p) = \frac{60p_1}{3p_1+40} + \frac{40p_2}{p_1+6p_2} - 20$
$e_2(p) = \frac{240p_2}{p_1+6p_2} + \frac{800}{p_2+60} - 40$

(f) $e_1(p) = \frac{60p_1(p_1+6p_2)+40p_2(3p_1+40)-20(p_1+6p_2)(3p_1+40)}{(p_1+6p_2)(3p_1+40)} = \left[3 - \frac{80}{p_1} - \frac{20}{p_2}\right]\frac{40p_1p_2}{(p_1+6p_2)(3p_1+40)}$
$e_2(p) = \frac{240p_2(p_2+60)+800(p_1+6p_2)-40(p_2+60)(p_1+6p_2)}{(p_2+60)(p_1+6p_2)} = \left[-\frac{40}{p_2} + \frac{120}{p_1} - 1\right]\frac{40p_1p_2}{(p_2+60)(p_1+6p_2)}$
Markets cleat at $p_1 = 40$ and $p_2 = 20$. The third market will clear due to Walras Law.
The model could be analyzed without linear approximations:
$e_1 > 0$ if $\frac{1}{p_2} < \frac{1}{20}(3 - 80\frac{1}{p_1})$ $\qquad e_2 > 0$ if $\frac{1}{p_2} > \frac{1}{40}(120\frac{1}{p_1} - 1)$
(g) $e_1'(p)|_{\bar{p}} = \left[\frac{2400}{(3p_1+40)^2} - \frac{40p_2}{(p_1+6p_2)^2}, \frac{40p_1}{(p_1+6p_2)^2}\right]\Big|_{(40,20)} = \left[\frac{1}{16}, \frac{1}{16}\right]$
Hence, $\bar{e}_1(p) = \frac{1}{16}(p_1 - 40) + \frac{1}{16}(p_2 - 20)$
(h) $e_2'(p)|_{\bar{p}} = \left[-\frac{240p_2}{(p_1+6p_2)^2}, \frac{240p_1}{(p_1+6p_2)^2} - \frac{800}{(p_2+60)^2}\right]\Big|_{(40,20)} = \left[-\frac{3}{16}, \frac{1}{4}\right]$
Hence, $\bar{e}_2(p) = -\frac{3}{16}(p_1 - 40) + \frac{1}{4}(p_2 - 20)$
(i,j) The equilibrium is unstable.



Exercise 3

$U^A(x) = 3\ln(x_1 - 4) + \ln(x_2 - 4) \qquad U^B(x) = \ln(x_1 - 4) + 3\ln(x_2 - 4)$

(a) $z_i^j = x_i^j - 4 \qquad \Rightarrow \qquad U^A(x) = 3\ln z_1 + \ln z_2 \qquad U^B(x) = \ln z_1 + 3\ln z_2$

Cobb-Douglas demands: $\qquad z^A(p) = [\frac{3I^A}{4p_1}, \frac{I^A}{4p_2}] \qquad z^B(p) = [\frac{I^B}{4p_1}, \frac{3I^B}{4p_2}]$, where

$\qquad I^A = p_1(\bar{x}_1^A - 4) + p_2(\bar{x}_2^A - 4) \qquad I^B = p_1(\bar{x}_1^B - 4) + p_2(\bar{x}_2^B - 4)$

Hence, $x_1^A(p) = 4 + \frac{3(p_1(\bar{x}_1^A-4)+p_2(\bar{x}_2^A-4))}{4p_1}$, $x_1^B(p) = 4 + \frac{p_1(\bar{x}_1^B-4)+p_2(\bar{x}_2^B-4)}{4p_1}$

(b) $x_1(p) = x_1^A(p) + x_1^B(p) = 8 + \frac{3}{4}(\bar{x}_1^A - 4) + \frac{1}{4}(\bar{x}_1^B - 4) + \frac{p_2}{p_1}\left[\frac{3}{4}(\bar{x}_2^A - 4) + \frac{1}{4}(\bar{x}_2^B - 4)\right]$

(b) Thinking in coordinates z the solution to the Pareto problem is this:

$L = 3\ln z_1 + \ln z_2 + \lambda(U - \ln(4 - z_1) - 3\ln(4 - z_2))$

FOC: $\qquad \frac{3(4-z_1)}{z_1} = \lambda = \frac{4-z_2}{3z_2} \qquad \Leftrightarrow \qquad \frac{9}{z_1} = 2 + \frac{1}{z_2}$

Find the price ratios that could possibly support these PE allocations: $\qquad \frac{p_1}{p_2} = \frac{3/z_1}{1/z_2} = \frac{3z_2}{z_1}$

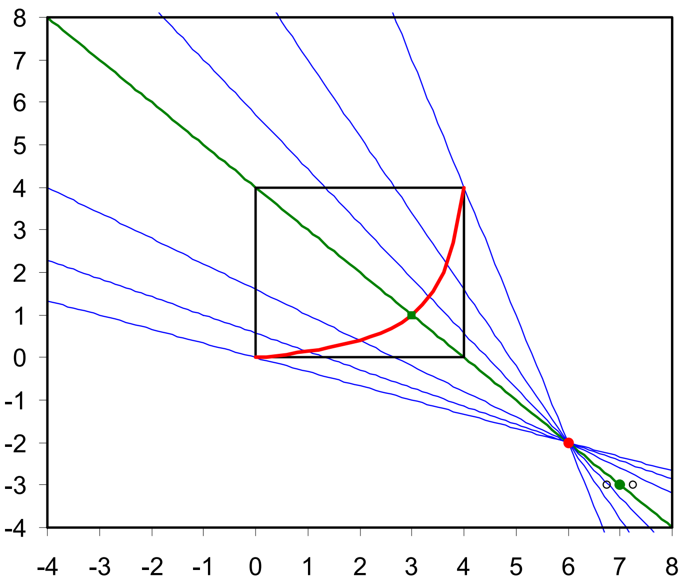
Draw a line through $(z_1, z_2) = (z_1, \frac{1}{\frac{9}{z_1}-2})$ with slope $-\frac{3z_2}{z_1} = -\frac{3}{9-2z_1}$:

$\qquad y_2 = -\frac{3}{9-2z_1}(y_1 - z_1) + \frac{z_1}{9-2z_1} = \frac{4z_1-3y_1}{9-2z_1}$

Given endowment point: (6,-2) - must be on some of these lines, because any Walrasian Equilibrium is Pareto Efficient due to 1st Welfare Theorem and we have found all supporting trade-lines.

$-2 = -\frac{3}{9-2z_1}(6 - z_1) + \frac{z_1}{9-2z_1} \qquad \Leftrightarrow \qquad -2(9 - 2z_1) = -18 + 4z_1$, which holds for any z_1.

(c) The set of WE price-ratios is given by $\left.\frac{3}{9-2z_1}\right|_{(0,4)} = (\frac{1}{3}, 3)$



(d,e,f) Given endowment (7,-3) plug into trade-lines:

$-3(9 - 2z_1) = 4z_1 - 21 \qquad \Leftrightarrow \qquad z_1 = 3 \qquad$ only one WE with $p_1/p_2 = \frac{3}{9-2z_1} = 1$.

$-3 = \frac{4z_1-3y_1}{9-2z_1} \Rightarrow z_1(y_1) = (27 - 3y_1)/2, \qquad z_2(y_1) = [\frac{9}{z_1(y_1)} - 2]^{-1} \qquad p_1/p_2 = \frac{3}{9-2z_1(y_1)} = \frac{3}{3y_1-18}$

Hence Alex's consumption of both goods is a decreasing function in Alex's endowment of commodity 1. And the price ratio is a decreasing function in Alex's endowment of commodity 1.

Alex is better off in equilibrium by giving Bev some of his endowment of good 1.

If he gave Bev $\frac{2}{3}(-\varepsilon)$ of good 1, he would obtain the best possible outcome: (4,4) in terms of z.

Exercise 4

(a) $L = r_1 z_1 + r_2 z_2 + \lambda(\sqrt{z_1 z_2/3} - q_A)$

$\frac{1}{\lambda} = \frac{1}{2r_1}\sqrt{\frac{z_2}{3z_1}} = \frac{1}{2r_2}\sqrt{\frac{z_1}{3z_2}} \quad \Rightarrow z_1 r_1 = z_2 r_2 = C/2 \quad \Rightarrow \quad C_A(q_A) = 2\sqrt{3r_1 r_2} * q_A$

$q_B = \frac{2}{3}\min[z_1, z_2] \quad \Rightarrow \quad z_1 = z_2 = \frac{3}{2}q_B$ in optimum $\Rightarrow \quad C_B(q_B) = \frac{3}{2}(r_1 + r_2)q_B$

Unit costs: $c_A = 2\sqrt{3r_1 r_2}, \quad c_B = \frac{3}{2}(r_1 + r_2)$

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(b) In equilibrium $MC_A = p_A$, $MC_B = p_B \Rightarrow \frac{3}{2}(r_1 + r_2) = 1 = 2\sqrt{3r_1 r_2}$.

Hence, $1 = 12(\frac{2}{3} - r_1)r_1 \Rightarrow 12r^2 - 8r + 1 = 0 \Rightarrow r_1 = \{\frac{1}{6}, \frac{1}{2}\} \Rightarrow r_2 = \{\frac{1}{2}, \frac{1}{6}\}$

Case 1: $r = (\frac{1}{2}, \frac{1}{6}) \Rightarrow z_A = (\frac{1}{2r_1}, \frac{1}{2r_2}) = (1, 3)$, $z_B = (\frac{3}{2}, \frac{3}{2})$

Case 2: $r = (\frac{1}{6}, \frac{1}{2}) \Rightarrow z_A = (\frac{1}{2r_1}, \frac{1}{2r_2}) = (3, 1)$, $z_B = (\frac{3}{2}, \frac{3}{2})$

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(c) Case 1:

$\{q_A + \frac{3}{2}q_B = 300, 3q_A + \frac{3}{2}q_B = 200\}$, Solution is: $[q_A = -50, q_B = \frac{700}{3}]$

Case 2:

$\{3q_A + \frac{3}{2}q_B = 300, q_A + \frac{3}{2}q_B = 200\}$, Solution is: $[q_A = 50, q_B = 100]$

Hence, only case 2 is an equilibrium.

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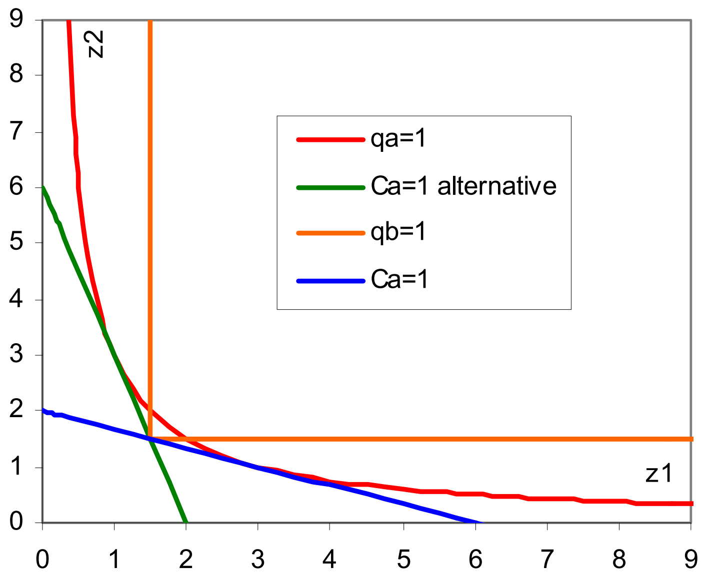
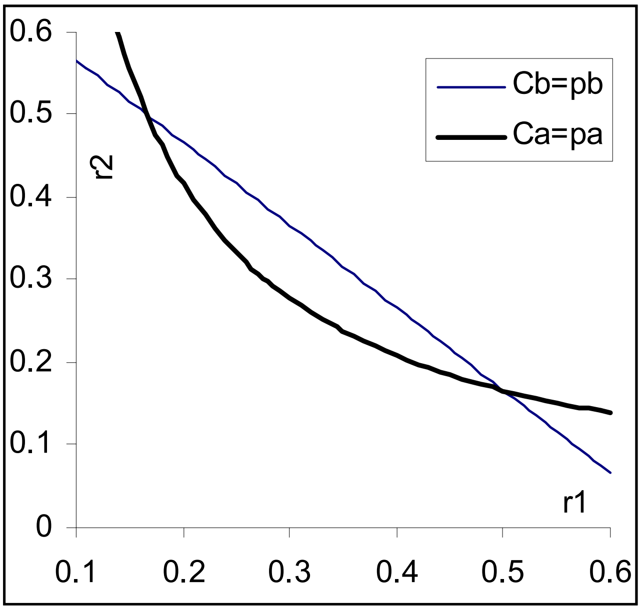
(d) Unit isoquants: A: $z_1 z_2 = 3 \qquad$ B: $\{z_1 \geq \frac{3}{2}, z_2 = \frac{3}{2}\} \cup \{z_1 = \frac{3}{2}, z_2 \geq \frac{3}{2}\}$

Unit isocosts: A: $z_1 r_1 + z_2 r_2 = 1 \qquad$ B: $z_1 r_1 + z_2 r_2 = 1$

The unit cost line must be tangent to the unit isoquant because that's a solution to a maximization problem: minimize cost given quantity or maximize quantity given cost:



(e) From the graph it is possible to see why there could be at least two solutions. There could be two lines, tangent to both graphs of unit costs. Hence, they both solve the problem, and we have at least two pairs of input prices, equating marginal costs to output prices. If we use both relationships and depict them in the input-price space (r_1, r_2) the solutions are easily figured out:



(f) Input prices will not change because in the CRS world the relationship between output prices and input prices does not depend on the endowment.

Case 1: $\{q_A + \frac{3}{2}q_B = 300, 3q_A + \frac{3}{2}q_B = 250\}$, Solution is: $\left[q_A = -25, q_B = \frac{650}{3}\right]$
Case 2: $\{3q_A + \frac{3}{2}q_B = 300, q_A + \frac{3}{2}q_B = 250\}$, Solution is: $[q_A = 25, q_B = 150]$
Hence, only case 2 is an equilibrium.

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(g) Case 1: $\{q_A + \frac{3}{2}q_B = 300, 3q_A + \frac{3}{2}q_B = 400\}$, Solution is: $\left[q_A = 50, q_B = \frac{500}{3}\right]$
Case 2: $\{3q_A + \frac{3}{2}q_B = 300, q_A + \frac{3}{2}q_B = 400\}$, Solution is: $[q_A = -50, q_B = 300]$
Hence, only case 1 is an equilibrium.