Handout for TA section 2

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Solow-Swan Model

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Production function: $Y = AK^{\alpha}L^{1-\alpha}$ (A - technology, K - capital, L - labor)
Capital accumulation: $\dot{K} = S - D = sY - \delta K$ (savings - depreciation)
Technological progress: $\dot{A} = g_A A$ (technology grows at a constant rate)
Labor force: $\dot{L} = g_L L$ (number of workers grows at a constant rate)

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Step 1.
Definition of steady-state: all variables grow at constant rates
$\frac{\dot{Y}}{Y} = const1 \quad \frac{\dot{K}}{K} = const2 \quad \frac{\dot{L}}{L} = const3 \quad \frac{\dot{A}}{A} = const4$
If there exists a steady state, it must be true that:
$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta = const \quad \Rightarrow \quad \frac{Y}{K} = const$

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Step 2:
Therefore, $\ln\frac{Y}{K} = \ln Y - \ln K = const$
Hence, $\frac{d}{dt}(\ln Y - \ln K) = 0 \quad \Leftrightarrow \quad \frac{d\ln Y}{dt} = \frac{d\ln K}{dt} \quad \Leftrightarrow \quad \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K}$
If there exists a steady-state, capital and GDP grow at the same rate in it.
Let's call this rate g : $\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = g$

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Step 3:
Now we need to find this growth rate. To do that we use the production function:
$Y = AK^{\alpha}L^{1-\alpha} \quad \Leftrightarrow \quad \ln Y = \ln A + \alpha\ln K + (1-\alpha)\ln L$
Hence, $\frac{d\ln Y}{dt} = \frac{d\ln A}{dt} + \alpha\frac{d\ln K}{dt} + (1-\alpha)\frac{d\ln L}{dt}$
Therefore, $\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}$.
Replacing the growth rates we get:
$g = g_A + \alpha g + (1-\alpha)g_L \quad \Leftrightarrow \quad (1-\alpha)g = g_A + (1-\alpha)g_L$
Hence, $g = \frac{g_A}{1-\alpha} + g_L$. This is the steady-state growth rate of capital and output.

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Notice, that, since $\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta = g = \frac{g_A}{1-\alpha} + g_L$, we can find $\frac{Y}{K} = \frac{1}{s}\left(\frac{g_A}{1-\alpha} + g_L + \delta\right)$

Also, GDP per capita grows at a constant positive rate, defined only by the speed of technological change: $d\ln\frac{Y}{L} = d\ln Y - d\ln K = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \frac{g_A}{1-\alpha} + g_L - g_L = \frac{g_A}{1-\alpha} > 0$