Econ202B TA section 9 Anton Cheremukhin

March 7, 2008

Political Competition

General framework:

An individual chooses a consumption bundle so as to maximize his utility $U(c_i, q, p, \alpha^i)$ subject to (budget or time) constraint $H(c_i, p, q, \alpha^i) \geq 0$.

q - policy variable

α^i - type of agent, distributed according to $F()$ and has mean α

c_i - consumption choice of the agent

p - some market-determined variables

Indirect utility: $\tilde{W}(q, p, \alpha^i) = \max_{c_i} \{U(c_i, q, p, \alpha^i) | H(c_i, p, q, \alpha^i) \geq 0\}$

The policymaker has to respect constraints connecting his policy and the market variables: $G(q, p) \geq 0$

When it's binding, assume that $G(q, p) = 0 \quad \Leftrightarrow \quad p = P(q)$

Then $W(q, \alpha^i) = \tilde{W}(q, P(q), \alpha^i)$ – reduced-form policy preferences.

Preferred policy for agent i: $q(\alpha^i) = \arg\max_q W(q, \alpha^i)$.

Definition: Preferences of type α^i are single-peaked if:

if $q'' < q' < q(\alpha^i) \quad \Rightarrow \quad W(q'', \alpha^i) < W(q', \alpha^i)$

if $q'' > q' > q(\alpha^i) \quad \Rightarrow \quad W(q'', \alpha^i) < W(q', \alpha^i)$

Definition: A Condorcet winner is a policy q that beats any other feasible policy in a pairwise vote.

Assumptions:

1) direct democracy: citizens themselves make policy choices

2) Sincere voting: in any comparison each citizen votes for the policy that gives him higher utility

3) Open agenda: Citizens vote in rounds over pairs of policy alternatives, and the winner is carried to next round.

Proposition: if preferences are single-peaked for a given ordering of policy alternatives, then a Condorcet winner always exists and coincides with the median voter.

Downsian Electoral Competition:

two parties: A, B.

Each party chooses a policy in order to maximize the probability of winning.

Timing:

1) both parties simultaneously noncooperatively announce policies

2) voters choose between parties

3) the winner implements the announced policy

A **Majority Voting Equilibrium** is a Nash equilibrium of this game, when the probability of agent i voting for party A and B respectively is:

$$p_A^i = \begin{cases} 0 & W(q_A, \alpha^i) < W(q_B, \alpha^i) \\ \frac{1}{2} & W(q_A, \alpha^i) = W(q_B, \alpha^i) \\ 1 & W(q_A, \alpha^i) > W(q_B, \alpha^i) \end{cases} \qquad p_B = 1 - p_A$$

.

A **Probabilistic Voting Equilibrium** is a Nash equilibrium of this game, when the probability of agent i voting for party A and B respectively is:

$p_A^i = F^i\left(W(q_A, \alpha^i) - W(q_B, \alpha^i)\right) \qquad p_B^i = 1 - p_A^i$

.

The probability of party A winning is the average of the probabilities of all agents:

$\pi_A = \frac{1}{N}\Sigma_{i=1}^N F^i\left(W(q_A, \alpha^i) - W(q_B, \alpha^i)\right) \to \max_{q_A}$

FOC: $\quad \Sigma_{i=1}^N F^{i\prime}\left(W(q_A, \alpha^i) - W(q_B, \alpha^i)\right) W_q'(q_A, \alpha^i) = 0$

Both parties will choose the same platform $q_A = q_B$, so $W(q_A, \alpha^i) = W(q_B, \alpha^i)$

FOC: $\quad \Sigma_{i=1}^N F^{i\prime}(0)\, W_q'(q_A, \alpha^i) = 0.$

So agents will be weighted by their marginal densities $F^{i\prime}(0)$ in the equilibrium point.

.

Example:

$W_i = c_i + \alpha^i V(y) \qquad s.t. \qquad c_i = 1 - q$

q - policy (tax)

α^i - type of agent, distributed according to $F()$ and has mean α

c_i - consumption choice of the agent

W_i - total utility of the agent of type i

$V(.)$ - concave well-behaved function

$q = y$ - budget constraint of the government

1) Show that preferences over policy are single-peaked.

$W_i = c_i + \alpha^i V(y) = 1 - q + \alpha^i V(y) = 1 - y + \alpha^i V(y)$

$W(y, \alpha^i) = 1 - y + \alpha^i V(y)$

$W_y'(y, \alpha^i) = -1 + \alpha^i V'(y) = 0 \qquad \Leftrightarrow \qquad y = V'^{-1}\left(\frac{1}{\alpha^i}\right)$

$W_y''(y, \alpha^i) = \alpha^i V''(y) < 0$

If $V()$ is continuous and concave, then W is continuous, concave and has a unique maximum.

Therefore, it is single-peaked. Hence the median voter is the Condorcet winner.

If you have two parties, and a continuum of voters, then the majority-voting equilibrium is:

$y = V'^{-1}\left(\frac{1}{\alpha^m}\right).$

The probabilistic voting equilibrium is determined by:

$\Sigma_{i=1}^N F^{i\prime}(0)\, \alpha^i V'(y) = \Sigma_{i=1}^N F^{i\prime}(0)$

$V'(y) = \frac{\Sigma_{i=1}^N F^{i\prime}(0)}{\Sigma_{i=1}^N F^{i\prime}(0)\alpha^i} \qquad \Rightarrow \qquad y = V'^{-1}\left(\frac{\Sigma_{i=1}^N F^{i\prime}(0)}{\Sigma_{i=1}^N F^{i\prime}(0)\alpha^i}\right)$

If all the functions are the same, then the outcome is determined by the mean voter, not the median.

So this probabilistic voting scheme achieves a utilitarian social optimum.

Algorithm to solve the computational question on the homework:

1) set a grid of tax rates τ_i

2) set a grid of corresponding probabilities $p_i = \frac{1}{2}\left(1 + \tau_i^2\right)$

3) set an initial guess $\Psi_0(p)$

outside loop until $\Psi_0(p) = \Psi_1(p)$

4) set initial guesses for value functions $v_{P0}(p), v_{R0}(p)$

inside loop until $v_{P0}(p) = v_{P1}(p), \qquad v_{R0}(p) = v_{R1}(p)$

5) compute $p'(\Psi_0(p))$

6) compute $v_{P1}(p)$ using y_P, $v_{P0}(p)$, $v_{R0}(p)$, $\Psi_0(p)$ and $p'(\Psi_0(p))$

7) compute $v_{R1}(p)$ using y_R, $v_{P0}(p)$, $v_{R0}(p)$, $\Psi_0(p)$ and $p'(\Psi_0(p))$

here it is useful to use a spline function

spline(x,y,xx) approximates the matrix-correspondence y(x) at the point xx (xx is in between grid points of x).

8) iterate inside loop untill convergence

end of inside loop

9) take a grid of τ^i

10) take the results v_{P1} and v_{R1}

11) for each value on that grid τ^i compute $\tilde{v}(p, \tau)$

12) for each p find the argmaximum of this function $\tau(p)$

13) this is your new function $\Psi_1(p) = \tau(p)$

14) iterate over Ψ until convergence

end of outside loop

15) plot the graps of v_R, v_P, Ψ.

16) find the optimal τ and p.