Exercise 12.1. *A small open economy (Razin and Sadka, 1995)*

Consider the non stochastic model with capital and labor in this chapter, but assume that the economy is a small open economy that cannot affect the international rental rate on capital, r_t^*. Domestic firms can rent any amount of capital at this price, and the households and the government can choose to go short or long in the international capital market at this rental price. There is no labor mobility across countries. We retain the assumption that the government levies a tax τ_t^n on households' labor income but households no longer have to pay taxes on their capital income. Instead, the government levies a tax $\hat{\tau}_t^k$ on domestic firms rental payments to capital regardless of the capital origin (domestic or foreign). Thus, a domestic firm faces a total cost of $(1+\hat{\tau}_t^k)r_t^*$ on a unit of capital rented in period t.

a. Solve for the optimal capital tax $\hat{\tau}_t^k$.

b. Compare the optimal tax policy of this small open economy to that of the closed economy of this chapter.

Solution

a. and **b.** The household problem is to choose consumption, labor, capital and bonds holding $\{c_t, n_t, k_{t+1}^H, b_{t+1}^H\}_{t=0}^{+\infty}$, so as to maximize

$$\sum_{t=0}^{+\infty} \beta^t u(c_t, 1-n_t), \tag{129}$$

subject to

$$c_t + k_{t+1}^H + \frac{b_{t+1}^H}{R_t^*} = (1-\tau_t^n)w_t n_t + r_t^* k_t^H + (1-\delta)k_t^H + b_t^H, \tag{130}$$

and a transversality condition. No arbitrage imposes that $R_t^* = r_{t+1}^* + (1-\delta)$. The government has the budget constraint

$$g_t + b_t^G = \hat{\tau}_t^k r_t^* k_t + \tau_t^n w_t n_t + \frac{b_{t+1}^G}{R_t^*}, \tag{131}$$

where b_t^G denotes government debt and k_t is the total capital stock of the economy, that may not be entirely owned by domestic household. A transversality condition needs also to be added to the government budget constraint. Firm's profit maximization implies :

$$F_k(k_t, n_t) = (1+\hat{\tau}_t^k)r_t^* \tag{132}$$

$$F_n(k_t, n_t) = w_t. \tag{133}$$

Add (130) and (131), and use the homogeneity of degree one of the production function to obtain

(134)
$$c_t + k_{t+1} + g_t + (k^H_{t+1} - k_{t+1}) + (b^G_t - b^H_t) = F(k_t, n_t) + \frac{b^G_{t+1} - b^H_{t+1}}{R^*_t} + (1 - \delta + r^*_t)(k^H_t - k_t).$$

Observe that, since both the government and the households can borrow and save in the international capital market, the resource constraint of the close economy does not necessarily hold. First, b^G_t may be different from b^H_t which means that all government bonds are not necessarily owned by domestic households. $b^G_t - b^H_t$ may be interpreted as the country deficit. Similarly k_t may be different from k^H_t. If $k_t > k^H_t$ then some domestic capital is owned by foreign investors. If on the other hand $k_t < k^H_t$, then domestic investors own foreign capital.

The primal approach to the Ramsey problem in this small economy context is to maximize the representative agent utility subject to the intertemporal budget constraint of the household and the intertemporal budget constraint of the government. Observe that those two intertemporal budget constraints do not imply the resource constraint of the closed economy. To write these constraints, we note that the first order conditions of the household's problem give

$$\beta^t u_c(t) = \lambda_0 q^*_t \tag{135}$$
$$\beta^t u_l(t) = \lambda_0 q^*_t (1 - \tau^n_t) w_t, \tag{136}$$

where λ_0 is the multiplier on the time zero budget constraint and $1/q^*_t = R^*_{t-1} R^*_{t-2} \dots R^*_0$. The intertemporal budget constraint of the household becomes

$$\sum_{t=0}^{+\infty} \left(q^*_t c_t - \beta^t \frac{u_l(t)}{u_c(0)} n_t \right) = \left((1 - \delta) + r_0 (1 - \tau^k_0) \right) k_0 + b_0. \tag{137}$$

To write the government intertemporal budget constraint, we note that

$$\begin{aligned} \hat{\tau}^k_t r^*_t k_t + \tau^n_t w_t n_t &= (1 + \hat{\tau}^k_t) r^*_t k_t + w_t n_t - r^*_t k_t - (1 - \tau^n_t) w_t n_t \\ &= F(k_t, n_t) - r^*_t k_t - (1 - \tau^n_t) w_t n_t. \end{aligned}$$

So that the intertemporal budget constraint of the government is

$$b^G_0 + \sum_{t=0}^{+\infty} q^*_t \left(g_t - \hat{\tau}^k_t r^*_t k_t + \tau^n_t w_t n_t \right) = \tag{138}$$

$$b^G_0 + \sum_{t=0}^{+\infty} \left(q^*_t g_t - \beta^t \frac{u_l(t)}{u_c(0)} n_t + q^*_t F(k_t, n_t) - q^*_t r^*_t k_t \right) = 0. \tag{139}$$

The Ramsey problem is to choose $\{c_t, n_t, k_{t+1}\}$ to maximize (129) subject to (137) and (139). Observe that the capital stock appears only in the government's budget constraint. Taking derivative in the Lagrangian with respect to k_t yields

(140) $$F_k(t) = r_t^*.$$

which, together with (132) implies that $\hat{\tau}_t^k = 0$ for all $t \geq 1$.

Exercise 12.2. *Exercise 12.2 Consumption Taxes*

Consider the non stochastic model with capital and labor in this chapter, but instead of labor and capital taxation assume that the government sets labor and consumption taxes, $\{\tau_t^n, \tau_t^c\}$. Thus, the household's present-value budget constraint is now given by

$$\sum_{t=0}^{\infty} q_t^0 (1 + \tau_t^c) c_t = \sum_{t=0}^{\infty} q_t^0 (1 - \tau_t^n) w_t n_t + [r_0 + 1 - \delta] k_0 + b_0.$$

a. Solve for the Ramsey plan.

b. Suppose that the solution to the Ramsey problem converges to a steady state. Characterize the optimal limiting sequence of consumption taxes.

c. In the case of capital taxation, we imposed an exogenous upper bound on τ_0^k. Explain why a similar exogenous restriction on τ_0^c is needed to ensure an interesting Ramsey problem. (Hint: Explore the implications of setting $\tau_t^c = \tau^c$ and $\tau_t^n = -\tau^c$ for all $t \geq 0$, where τ^c is a large positive number.)

Solution

a. We follow the steps described in the paragraph "constructing the Ramsey Plan".

Step 1: Write the household's problem

We first recall the household problem when trading is sequential. The household chooses consumption, labor, capital and bond holdings $\{c_t, n_t, k_{t+1}, b_{t+1}\}_{t=0}^{+\infty}$ so as to maximize

(141) $$\sum_{t=0}^{+\infty} \beta^t u(c_t, 1 - n_t),$$

subject to

(142) $$(1 + \tau_t^c) c_t + k_{t+1} + \frac{b_{t+1}}{R_t} = (1 - \tau_t^n) w_t n_t + (r_t + (1 - \delta)) k_t + b_t.$$

Attach a Lagrange multiplier $\beta^t \lambda_t$ to time t budget constraint. The first order conditions of the household's problem are:

$$\begin{aligned} c_t: \quad \beta^t u_c(t) &= \lambda_t(1+\tau_t^c) \\ n_t: \quad \beta^t u_n(t) &= \lambda_t(1-\tau_t^n)w_t \\ k_{t+1}: \quad \lambda_t &= \beta\lambda_{t+1}\left[r_{t+1}+(1-\delta)\right] \\ b_{t+1}: \quad \frac{\lambda_t}{R_t} &= \beta\lambda_{t+1}. \end{aligned}$$

Observe that the previous equations imply the no-arbitrage condition

$$R_t = r_{t+1} + (1-\delta). \tag{143}$$

To apply the primal approach, we need to reformulate the household problem in the context of time zero trading. We thus define $1/q_t^0 \equiv R_{t-1}R_{t-2}\ldots R_0$. We have

$$\beta^t u_c(t) = \lambda_0 q_t^0 (1+\tau_t^c) \tag{144}$$

$$\beta^t u_n(t) = \lambda_0 q_t^0 (1-\tau_t^n) w_t. \tag{145}$$

Observe that (144) and (145) imply in particular that

$$\begin{aligned} q_t^0(1+\tau_t^c) &= \beta^t \frac{u_c(t)}{u_c(0)}(1+\tau_0^c) \\ q_t^0(1-\tau_t^n)w_t &= \beta^t \frac{u_l(t)}{u_c(0)}(1+\tau_0^c). \end{aligned}$$

Furthermore, (144) and (145) also show that the household choice of consumption and labor maximizes

$$\sum_{t=0}^{+\infty} \beta^t u(c_t, 1-n_t), \tag{146}$$

subject to an *intertemporal* budget constraint

$$\sum_{t=0}^{+\infty} q_t^0(1+\tau_t^c)c_t \leq \sum_{t=0}^{+\infty} q_t^0 w_t(1-\tau_t^n)n_t + \left(r_0+(1-\delta)\right)k_0 + b_0. \tag{147}$$

Step 2: Write the intertemporal budget constraint.

The intertemporal budget constraint is

$$\sum_{t=0}^{+\infty} \beta^t (1+\tau_0^c)(u_c(t)c_t - u_l(t)n_t) = u_c(0)\left((r_0+(1-\delta))k_0 + b_0\right).$$

Let $A \equiv u_c(0)\left((r_0+(1-\delta))k_0+b_0\right)$.

Step 3: Form the Lagrangian.

Now define:

$$V(c_t, n_t, \Phi) \equiv u(c_t, 1 - n_t) + \Phi(1 + \tau_0^c)(u_c(t)c_t - u_l(t)n_t).$$

The Lagrangian associated with the Ramsey plan is:

$$J = \sum_{t=0}^{+\infty} \beta^t \{V(c_t, n_t, \Phi) + \theta_t (F(k_t, n_t) - (1 - \delta)k_t - c_t - g_t - k_{t+1})\} - \Phi A.$$

The first order conditions are:

$$\begin{array}{lrcl} c_t: & V_c(t) & = & \theta_t, \quad t \geq 1 \\ n_t: & V_n(t) & = & -\theta_t F_n(t), \quad t \geq 1 \\ k_{t+1}: & \theta_t & = & \beta\theta_{t+1}[F_k(t+1) + (1 - \delta)], \quad t \geq 0 \\ c_0: & V_c(0) & = & \theta_0 + \Phi A_c \\ n_0: & V_n(0) & = & -\theta_0 F_n(0) + \Phi A_n. \end{array}$$

The Ramsey Plan is thus solution of the following system of difference equations:

$$\begin{array}{rcl} V_c(t) & = & \beta V_c(t+1)[F_k((t+1) + 1 - \delta], \quad t \geq 1 \\ V_n(t) & = & -V_c(t)F_n(t), \quad t \geq 1 \\ V_n(0) & = & [\Phi A_c - V_c(0)]F_n(0) + \Phi A_n. \end{array}$$

$$\begin{array}{l} c_t + g_t + k_{t+1} = F(k_t, n_t) + (1 - \delta)k_t \\ \sum_{t=0}^{+\infty} \beta^t (1 + \tau_0^c)(u_c(t)c_t - u_l(t)n_t) - A = 0. \end{array}$$

b. Assume this system converges to a steady state. Remember that the no-arbitrage condition (143) must hold, that is $R_t = \frac{q_t^0}{q_{t+1}^0} = (1 - \tau_{t+1}^k)F_k(t+1) + 1 - \delta$. The steady state version of the first difference equation defining the Ramsey plan is $1 = \beta\big[(1 - \tau_{t+1}^k)F_k(t+1) + 1 - \delta\big]$. So that:

$$\frac{q_t^0}{q_{t+1}^0} = \frac{1}{\beta}.$$

On the other hand, the first order conditions of the household problem gives:

$$\frac{q_t^0(1 + \tau_t^c)}{q_{t+1}^0(1 + \tau_{t+1}^c)} = \frac{u_c(t)}{\beta u_c(t+1)}.$$

In steady state, this becomes:

$$\frac{(1 + \tau_t^c)}{(1 + \tau_{t+1}^c)} = 1.$$

Which proves that the steady state consumption tax is constant.

c. Consider the household problem under the suggested taxation scheme:

$$\max_{\{c_t\}_{t=0}^{+\infty},\{l_t\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \beta^t u(c_t, 1-n_t)$$
subject to
$$\sum_{t=0}^{+\infty} q_t^0 (1+\tau^c) c_t = \sum_{t=0}^{+\infty} q_t^0 (1+\tau^c) w_t n_t + \left(r_0 + (1-\delta)\right) k_0 + b_0.$$

The budget set is more conveniently described as:

$$\sum_{t=0}^{+\infty} q_t^0 (c_t - w_t n_t) = \frac{(r_0 + (1-\delta))k_0 + b_0}{1+\tau^c}.$$

So that it is now apparent that this taxation scheme is equivalent to a lump sum tax on time 0 assets (capital and bonds) and and no other tax afterwards. As we know, this scheme is optimal because it does not induce distortion. Therefore, in order to study what an optimal distortive tax on consumption would be, we need to impose an upper bound on τ_0^c.