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February 15, 2008

Ramsey taxation

Household: $\Sigma_{t=0}^{\infty}\beta^t u(c_t, 1-n_t) \to \max_{k_{t+1}, b_{t+1}, c_t, n_t}$ (HH)

s.t. $(1-\tau_t^n) w_t n_t + \left(1-\tau_t^k\right) r_t k_t + b_t \geq c_t + k_{t+1} - (1-\delta) k_t + \frac{b_{t+1}}{R_{t+1}}$ (BC)

FOC: $u_{ct} = \lambda_t$ $u_{nt} = \lambda_t w_t (1-\tau_t^n)$

$\lambda_t = \beta\lambda_{t+1}\left[\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right]$ $\lambda_t = \beta\lambda_{t+1} R_{t+1}$

This is equivalent to:

$$u_{nt} = u_{ct} w_t (1-\tau_t^n) \quad (1)$$
$$u_{ct} = \beta u_{ct+1}\left[\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right] \quad (2)$$
$$\left[\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right] = R_{t+1} \quad (3)$$

Firm: $f(k_t, n_t) - w_t n_t - r_t k_t \to \max_{k_t, n_t}$ (F)

$$f_{kt} = r_t \quad (4)$$
$$f_{nt} = w_t \quad (5)$$

from CRS: $f_{kt}k_t + f_{nt}n_t = f(k_t, n_t)$

Resource Constraint: $f(k_t, n_t) \geq c_t + k_{t+1} - (1-\delta) k_t + g_t$ (RC)

Budget Constraint: $w_t n_t + r_t k_t \geq c_t + k_{t+1} - (1-\delta) k_t + \left[\frac{b_{t+1}}{R_{t+1}} - b_t + \tau_t^n w_t n_t + \tau_t^k r_t k_t\right]$ (BC)

Government Constraint: $g_t = \frac{b_{t+1}}{R_{t+1}} - b_t + \tau_t^n w_t n_t + \tau_t^k r_t k_t$ (GC)

A **competitive equilibrium** is an:

allocation $\{k_t, g_t, c_t, n_t\}_{t=0}^{\infty}$ price system $\{r_t, R_t, 1, w_t\}_{t=0}^{\infty}$ policy $\left\{b_t, \tau_t^k, \tau_t^n\right\}_{t=0}^{\infty}$

s.t.

1) given price and policy the allocation solves (HH) s.t. (BC) and solves (F)

2) given allocation and price, policy satisfies (GC)

3) allocation is feasible (RC)

A CE is characterized by 7 equations $(1, 2, 3, 4, 5, RC, GC \Leftrightarrow BC)$.

There are **several approaches** you could think of:

1) choose prices and policy in such a way, that the equilibrium that arizes maximizes welfare

2) choose prices, policy and allocations in such a way, that maximizes welfare given equilibrium conditions

3) choose allocations in such a way that maximizes welfare given they can be implemented as an equilibrium

We shall here discuss the third approach.

Solution using an implementability constraint

In the Ramsey problem the total resources thrown into the ocean by the government are given. k_0 is also given.

If you choose allocations $\{k_{t+1}, c_t, n_t\}_{t=0}^{\infty}$, then equation (4) pins down r_t, (5) pins down w_t, (1) pins down τ_t^n, (2) pins down τ_{t+1}^k, (3) pins down R_{t+1}. We are left with the choice of b_{t+1}, b_0, τ_0^k and equations (RC) and (BC) they have to satisfy.

We shall aggregate them into an implementability constraint:

Take (BC), multiply by $\beta^t u_{ct}$ and sum over t from 0 to T.

$\beta^t u_{ct} \times \left[(1-\tau_t^n) w_t n_t - c_t + \left(1-\tau_t^k\right) r_t k_t + (1-\delta) k_t - k_{t+1} + b_t - \frac{b_{t+1}}{R_{t+1}}\right] \geq 0$

$\Sigma_{t=0}^{T} \beta^t \left[u_{ct}\left[(1-\tau_t^n) w_t n_t - c_t\right] + u_{ct}\left\{\left[\left(1-\tau_t^k\right) r_t + 1 - \delta\right] k_t - k_{t+1}\right\} - u_{ct}\left[\frac{b_{t+1}}{R_{t+1}} + b_t\right]\right] \geq 0$

Remember that from (2&3): $\quad u_{ct-1} = \beta u_{ct}\left[\left(1-\tau_t^k\right) r_t + 1 - \delta\right] \quad \frac{u_{ct}}{R_{t+1}} = \beta u_{ct+1}$

Therefore, all the terms except the first ones are eliminated. Taking the limit over T and using the transfersality condition we get:

$\Sigma_{t=0}^{\infty} \beta^t u_{ct}\left[(1-\tau_t^n) w_t n_t - c_t\right] + u_{c0}\left[\left[\left(1-\tau_0^k\right) f_{k0} + 1 - \delta\right] k_0 - b_0\right] \geq 0$

Also, notice, that $u_{nt} = u_{ct} w_t (1-\tau_t^n)$. Therefore:

$$\Sigma_{t=0}^{\infty} \beta^t \left[u_{nt} n_t - u_{ct} c_t\right] + u_{c0}\left[\left[\left(1-\tau_0^k\right) f_{k0} + 1 - \delta\right] k_0 - b_0\right] \geq 0 \qquad (IMP)$$

Now, if we choose b_0 or τ_0^k we can pin down the other one and the whole sequence of bonds, using the government constraint (GC). So we can pin down all we need using all the constraints, except (RC) and (IMP), which also have to hold for there to be an equilibrium. Therefore, the planner should maximize welfare subject to (RC) and (IMP). In that case he can guarantee that the allocations he chooses are supported by a competitive equilibrium: he can find a unique price system and policy which satisfy all the conditions required for equilibrium.

$L = \Sigma_{t=0}^{\infty} \beta^t u(c_t, 1-n_t) - \lambda\left\{\Sigma_{t=0}^{\infty} \beta^t \left[u_{nt} n_t - u_{ct} c_t\right] + u_{c0}\left[\left[\left(1-\tau_0^k\right) f_{k0} + 1 - \delta\right] k_0 - b_0\right]\right\} +$
$+\Sigma_{t=0}^{\infty} \beta^t \mu_t \left[f(k_t, n_t) - c_t - k_{t+1} + (1-\delta) k_t - g_t\right] =$
$= \Sigma_{t=0}^{\infty} \beta^t \left[u(c_t, 1-n_t) - \lambda u_{nt} n_t + \lambda u_{ct} c_t\right] - \lambda u_{c0}\left[\left[\left(1-\tau_0^k\right) f_{k0} + 1 - \delta\right] k_0 - b_0\right] +$
$+\Sigma_{t=0}^{\infty} \beta^t \mu_t \left[f(k_t, n_t) - c_t - k_{t+1} + (1-\delta) k_t - g_t\right] \to \max_{c_t, n_t, k_{t+1}}$

Define $W_t = u(c_t, 1-n_t) - \lambda u_{nt} n_t + \lambda u_{ct} c_t$.Then,

$$\left[\begin{array}{c} \Sigma_{t=0}^{\infty} \beta^t W_t - \lambda u_{c0}\left[\left[\left(1-\tau_0^k\right) f_{k0} + 1 - \delta\right] k_0 - b_0\right] + \\ +\Sigma_{t=0}^{\infty} \beta^t \mu_t \left[f(k_t, n_t) - c_t - k_{t+1} + (1-\delta) k_t - g_t\right] \end{array}\right] \to \max_{c_t, n_t, k_{t+1}, b_0}$$

$\text{FOC}_{c_t}: \quad W_{ct} = \mu_t \quad \forall t > 0$
$\text{FOC}_{c_0}: \quad W_{c0} = \mu_0 + \lambda u_{cc0}\left[\left[\left(1-\tau_0^k\right) f_{k0} + 1 - \delta\right] k_0 - b_0\right]$
$\text{FOC}_{n_t}: \quad W_{nt} + \mu_t f_{nt} = 0 \quad \forall t > 0$
$\text{FOC}_{n_0}: \quad W_{n0} + \mu_0 f_{n0} + \lambda u_{cn0}\left[\left[\left(1-\tau_0^k\right) f_{k0} + 1 - \delta\right] k_0 - b_0\right] + \lambda u_{c0}\left(1-\tau_0^k\right) f_{kn0} k_0 = 0$
$\text{FOC}_{k_{t+1}}: \quad \mu_t = \beta \mu_{t+1}\left[f_{kt+1} + 1 - \delta\right]$

Define: $H = u_{c0}\left[\left[\left(1-\tau_0^k\right) f_{k0} + 1 - \delta\right] k_0 - b_0\right]$. Then the equations for the initial period are equivalent to: $\quad W_{n0} - \lambda H_n + \mu_0 f_{n0} = 0, \quad W_{c0} - \lambda H_c = \mu_0 \quad \Rightarrow$

$$\left[W_{n0} - \lambda H_n\right] + f_{n0}\left[W_{c0} - \lambda H_c\right] = 0$$
$$W_{nt} + W_{ct} f_{nt} = 0$$
$$W_{ct} = \beta W_{ct+1}\left[f_{kt+1} + 1 - \delta\right]$$
$$\text{where } W_t = u(c_t, 1-n_t) - \lambda u_{nt} n_t + \lambda u_{ct} c_t, \qquad H = u_{c0}\left[\left[\left(1-\tau_0^k\right) f_{k0} + 1 - \delta\right] k_0 - b_0\right]$$

These allow us to solve for $\{k_{t+1}, c_t, n_t\}_{t=0}^{\infty}, \lambda, b_0$ from which we can then back up all the equilibrium allocations, prices and policies.

Properties of the solution

On a steady-state we get: should have $W_{ct} = W_{ct+1}$. Therefore, $1 = \beta\left[f_{kt+1} + 1 - \delta\right]$. Remember that in equilibrium $u_{ct} = \beta u_{ct+1}\left[\left(1-\tau_{t+1}^{k}\right) r_{t+1} + 1 - \delta\right]$, which in steady-state yields: $1 = \beta\left[\left(1-\tau_{t+1}^{k}\right) r_{t+1} + 1 - \delta\right]$. Also in equilibrium $f_{kt+1} = r_{t+1}$. Hence in the optimal equilibrium it has to be the case that $1 - \tau_{t+1}^{k} = 1$ which means a zero tax rate on capital revenues. Therefore the government should only use public debt and a labor tax to finance its expenditures:

$$\boxed{\tau_t^k = 0}$$

Let's find the tax rate on labor. Equilibrium implies that $u_{nt} = u_{ct} w_t \left(1-\tau_t^n\right) = u_{ct} f_{nt}\left(1-\tau_t^n\right)$. Therefore, the labor tax is constant in steady-state and equal to:

$$\boxed{1-\tau_t^n = \frac{\frac{\partial u(c_t, 1-n_t)}{\partial n_t}}{\frac{\partial u(c_t, 1-n_t)}{\partial c_t}\frac{\partial f(k_t, n_t)}{\partial n_t}}}$$