

Problem Set 3

Ramsey Taxation

Question 1:

Solve Exercise 15.3 in Ljungqvist and Sargent.

Question 2:

Solve Exercise 15.5 in Ljungqvist and Sargent.

Question 3:

Consider the following variation on the Ramsey model. The economy lasts for two periods, $t = 0, 1$. Government consumption is exogenous. There is no government consumption in period 0: $g_0 = 0$. In the first period government consumption is random: $g_1 = 0$ with probability π , and $g_1 = G > 0$ with probability $1 - \pi$. There is no government debt outstanding at the beginning of period 0. Output is produced with a linear technology that uses labor as the only input. The resource constraint is therefore:

$$c_i + g_i = n_i.$$

The preferences of the representative household are:

$$u(c_0) + v(1 - n_0) + \beta E[u(c_1) + v(1 - n_1)],$$

where u and v are twice continuously differentiable, strictly increasing, and strictly concave, and $0 < \beta < 1$. The government has to finance its expenditure using a linear tax τ on labor income. Let τ_0 , $\tau_{1,0}$, and $\tau_{1,G}$ denote the tax rates in each period and state of the world.

- (a) Formulate the consumer's problem in the economy with distorting taxes and characterize the solution.
- (b) Formulate the implementability constraint for the benevolent Ramsey government. What other constraints does the government face?
- (c) Derive the conditions for a maximum of the government's problem.
- (d) Show that under the Ramsey policy government revenue is positive in each period and state of the world.
- (e) Briefly describe the policy for government debt that accompanies the Ramsey tax policy.

Question 4:

In this question, we are considering the Ramsey taxation problem in an environment where capital is the only source of income. The representative consumer solves:

$$\max \left\{ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right\}$$

subject to:

$$\sum_{t=0}^{\infty} q_t(c_t + x_t) \leq \sum_{t=0}^{\infty} q_t(1 - \tau_t)r_t k_t,$$
$$k_{t+1} = (1 - \delta)k_t + x_t,$$

with k_0 given. The representative firm operates the production function:

$$F(k) = Ak.$$

The government finances an exogenous stream of expenditures $\{g_t\}_{t=0}^{\infty}$ using the linear capital income tax τ_t . The government does not have to balance its budget every period, and can issue one-period government bonds.

- (a) Formulate the consumer's problem in the economy with distorting taxes and characterize the solution.
- (b) Formulate the implementability constraint for the benevolent Ramsey government. What other constraints does the government face?
- (c) Derive the conditions for a maximum of the government's problem.
- (d) Suppose there is a steady state. Determine whether the capital income tax is equal to zero in the steady state.
- (e) How does your answer change if the government is not able to issue bonds, and thus has to balance its budget every period?