Economics 202
Macroeconomics

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Problem Set 2

Dynamic Insurance

Question 1:
Solve Exercise 19.1 in Ljungqvist and Sargent.

Question 2:
Solve Exercise 19.4 in Ljungqvist and Sargent (the exercise refers to the analysis in Section 19.5 of the book).

Question 3:
This question asks you to compute an explicit solution to a two-period dynamic insurance problem. We will work with specific utility functions that lead to an analytic solution.

There is a continuum of agents of mass one. In each of the two periods, the aggregate endowment is equal to one. In period one, half the population gets income $y_1 = 0$ and the other half gets income $y_1 = 2$. In the second period, all agents get income $y_2 = 1$ for sure. The income realization in the first period is private information. There is no possibility of transferring resources between the two periods.

The utility function in the first period is given by:

$$u_1(c) = \begin{cases} c & \text{for} \quad c > 1, \\ \log(c) + 1 & \text{for} \quad c \leq 1. \end{cases}$$

In the second period, the utility function is:

$$u_2(c) = \begin{cases} c & \text{for} \quad c > 1, \\ 2\left(c - \frac{1}{2}\right) & \text{for} \quad c \leq 1. \end{cases}$$

The mechanism consists of a transfer $\tau_1 \geq 0$ from the agents who report high income to those who report low income in the first period, and in the second period of a transfer $\tau_2 \geq 0$ from those who reported low income in the first period to those who reported high income. This structure of transfers already takes account of the fact that the aggregate endowment is constant in each period (so that transfers have to balance out period-by-period), that the point of insurance is to give more initial consumption to agents with low income in the first period, and that to ensure incentive compatibility the direction of transfers has to be reversed in the second period. The social planning problem of choosing the optimal incentive-compatible transfer scheme can be written as:

$$\max \left\{ \frac{1}{2}[u_1(\tau_1) + \beta u_2(1 - \tau_2)] + \frac{1}{2}[u_1(2 - \tau_1) + \beta u_2(1 + \tau_2)] \right\}$$

subject to the incentive constraint:

$$u_1(2 - \tau_1) + \beta u_2(1 + \tau_2) \geq u_1(2 + \tau_1) + \beta u_2(1 - \tau_2).$$

Notice that the relevant incentive constraint is here that of a rich agent (income 2 in the first period) pretending to be poor (and thus receiving a positive transfer in the first and a negative transfer in the second period).

(a) Solve for the optimal transfer scheme, and comment on how the discount factor β affects the degree of risk sharing in this economy.

(b) Consider the same problem in a modified environment in which the agents (in addition to receiving/paying transfers) have access to a credit market in which they can borrow and lend unlimited amounts at the fixed interest rate $R = \frac{1}{\beta}$. How does the possibility of credit-market access affect the extent to which insurance can be offered? (Hint: Start by deriving the present-value budget constraint for each agent, and consider how the report (i.e., the choice between the two transfer schemes) affects the budget constraint.)