Economics 202 Matthias Doepke
Macroeconomics Winter 2008

Problem Set 1

Occupational Choice in General Equilibrium

Consider an economy with two types of agents, skilled and unskilled, and three occupations: managers, skilled workers, and unskilled workers. There exists measure μ_S of skilled and measure μ_U of unskilled people, where $\mu_U > \mu_S$. skilled people can either be managers or skilled workers, whereas the unskilled can only be unskilled workers. Every firm needs one manager. If a manager manages S skilled and U unskilled workers, the output of that person's production unit is:

$$S^\alpha U^\beta,$$

where $\alpha + \beta < 1$. The utility function of an individual is:

$$\log c - v(h),$$

where $c > 0$ and $h \in \{0, 0.4, 0.7\}$. If a person stays at home, we have $h = 0$. For a worker of either skill we have $h = 0.4$, and for a manager we get $h = 0.7$.

Question 1:
(a) Choose a suitable commodity space for this economy. Specify the consumption possibility sets and the agents' utility functions.

(b) Specify the aggregate production possibility set Y and the resource constraint.

(c) State the First Welfare Theorem and show that its conditions are satisfied for this problem.

(d) Formulate the social planning problem for this economy.

(e) What can you say about the relative consumption of managers, skilled workers, and unskilled workers? What can you say about the relative wages of managers, skilled workers, and unskilled workers (i.e., the corresponding equilibrium prices)? Prove your claims.

(f) Impose conditions on the model parameters that ensure that there are unemployed skilled and unskilled people in equilibrium.

Uncertainty in General Equilibrium

Consider a one-period economy with N agents and M factories. Agents have preferences of the form

$$U(c, n) = u(c) - v(n)$$

over consumption c and labor supply n. The function $u : \mathbf{R}_+ \to \mathbf{R}$ is bounded, continuously differentiable, strictly increasing, and strictly concave, with $\lim_{c \to 0} u'(c) = \infty$. The function $v : \mathbf{R}_+ \to \mathbf{R}$ is bounded, continuously differentiable, strictly increasing, and strictly convex.

Each of the M firms operates a technology described by the production function

$$F_m(\epsilon, n) = \epsilon_m f(n).$$

Here the function $f : \mathbf{R}_+ \to \mathbf{R}_+$ is continuously differentiable, strictly increasing, and strictly concave, with $f(0) = 0$, $\lim_{n \to 0} f'(n) = \infty$, and $\lim_{n \to \infty} f'(k) = 0$. ϵ_m is a nonnegative productivity shock. The vector $\epsilon \equiv (\epsilon_1, \epsilon_2, \ldots, \epsilon_M)'$ can take L different values, and the probability of vector ϵ^l is given by $p(l)$. Obviously, we have $p(l) \geq 0$ and $\sum_{l=1}^{L} p(l) = 1$.

Question 2:
(a) Describe this economy in the style of the "Theory of Value." What is the commodity space? What is the consumption set? What is the aggregate production set? What is the resource constraint? (Hint: No lotteries are required here, but you need different commodities for each state of nature.)

(b) Define a competitive equilibrium and show that the conditions for the First and Second Welfare Theorem are satisfied.

(c) Since the Welfare Theorems are satisfied, we can solve for equilibrium allocations by solving a Pareto planning problem for the economy. Write down the Pareto problem and derive the first-order conditions. What can be inferred about the properties of an equilibrium? Among other things, make sure you determine whether the sharing rules for consumption and labor are linear. If your answer is no, which additional assumptions would ensure that the answer is yes? Prove your claims.

(d) Assume you have found an equilibrium allocation. Use this allocation to find an equilibrium price system.

Private Information in General Equilibrium

Consider the adverse-selection model considered in class. There is measure one of ex-ante identical agents. Ex-post, agents turn into one of two types, θ_1 and θ_2, with equal probability (i.e., each type makes up one-half of the population ex post). The type of each agent is private information. The utility function of the θ_1 type is:

$$U(c, \theta_1) = \sqrt{c},$$

while the θ_2 type has utility:

$$U(c, \theta_2) = c.$$

Each agents is endowed with one unit of the consumption good.

Question 3:
(a) Specify this economy in the language of Debreu. The commodities should be lotteries over a discrete grid for consumption, given by:

$$C = \left\{0, \frac{1}{4}, c_{\max}\right\}.$$

Describe the timing of this economy. When are commodities traded, and when does consumption take place relative to the revelation of information?

(b) Specify $c_{\max}$ such that the first-best allocation can be achieved in general equilibrium. Find an equilibrium allocation and price vector.

Computing Private Information Problems

Private information problems can often be conveniently computed as linear programs. In this problem set you will use linear programming to compute solutions to a variation of the adverse selection problem considered above.

As before, there is continuum of measure of of agents who are identical ex-ante, but turn into two different types ex-post, where each type makes up half of the population. However, unlike in the model considered above, both types exhibit risk aversion, albeit to a different degree. The two utility functions are:

$$U(c, \theta_1) = \frac{1}{8} \log c,$$

while the θ_2 type has utility:

$$U(c, \theta_2) = \sqrt{c}.$$

Question 4:
(a) Formulate the planning problem for this economy. The choice object for the planner should consist of probabilities x_n^i for agent of type i to receive consumption c_n, where c_n is an element of the finite grid C. Make sure that your optimization problem contains constraints that guarantee that the x_n^i are probabilities, a resource constraint, as well as incentive-compatibility (i.e., truth-telling) constraints.

(b) Solve the planning problem on a computer. Use the following grid for consumption values for your analysis:

$$C = \{0.01, 0.02, 0.03, \ldots, 10\}$$

Notice that your grid must not include zero consumption, since utility is undefined for this consumption value under logarithmic utility. Describe the optimal outcome: Does the planner use non-degenerate lotteries? What is the mean and variance of consumption of each type? Compute and compare the ex-ante utility of the agents under the first-best solution (i.e., under full information), under the information-constrained optimum (which you just computed), and under autarky (where each agent consumes the endowment).

Remarks on Computation:
Please submit the programs you used with your answer. You can use any software you want. Here is an overview of all commands that you need if you use Matlab:

`kron`(A, B)	$A \otimes B$ (Kronecker product). Useful for setting up constraints.
`eye`(n)	n-dimensional identity matrix.
`ones`(n, m)	$n \times m$ matrix of ones.
`zeros`(n, m)	$n \times m$ matrix of zeros.
`linspace`(min, max, n)	Row vector with n evenly spaced elements from min to max.

You also need a linear-programming routine to solve this problem. The Matlab function `linprog` is sufficient for this application. You call `linprog` like this:

```
x=Linprog(f,A,b,Aeq,beq,lb,ub)
```

The routine solves the problem $\min_x f'x$ subject to $Ax \leq b$ and $Aeq\ x = beq$. f, b and beq are vectors, A is the matrix of inequality constraints, and Aeq defines the equality constraints. lb is a vector of lower bounds for x (use zeros), and ub is a vector of upper bounds (use ones). Notice that the routine minimizes the objective function, and the constraints are written as less-than-or-equal-to. You will have to write your program so that it fits this formulation.