

Midterm

Dynamic Contracting

Consider the following environment. There is an agent with the utility function:

$$\sum_{t=0}^{\infty} \beta^t \log(c_t).$$

In every period, the agent receives an income shock y_t , where $y_t \in \{y_L, y_H\}$ and $y_L < y_H$. The income shock follows a i.i.d. process with probabilities $\pi_L > 0$ and $\pi_H > 0$. There is a social planner who provides transfers τ_t to the agent, and has preferences:

$$\sum_{t=0}^{\infty} \gamma^t (-\tau_t),$$

where $\gamma < \beta$.

Question 1:

(a) Assume that the income shock y_t is publicly observed. Set up the social planning problem of minimizing the cost of providing reservation utility w_0 to the agent, and solve the planning problem. How does consumption evolve over time?

(b) Now assume that the income shock y_t is private information. Set up the recursive planning problem for this case (just write down the problem; no further analysis required).

Externalities in General Equilibrium

Consider the following economy. There is measure one of ex-ante identical people. Ex post, people turn into one of two types, smokers (mass or probability S) and non-smokers (mass or probability N), where $S < N$ and $S + N = 1$. All consumption in this economy takes place in restaurants, where for simplicity we assume that each restaurant has space for exactly two guests. Every consumer has to be allocated to a restaurant in order to consume. Smokers impose a negative externality on the other guest of the restaurant that they visit. More specifically, preferences are given by:

$$u(c, G_S) = \ln(c) - v(G_S),$$

where c is consumption, v is an increasing function, and G_S is the number of smoking guests in the restaurant, where $G_S \in \{0, 1, 2\}$. Notice that we assume that smokers have a negative effect even on their own utility, because any restaurant that they visit will contain at least one smoker (smoking is not a choice variable, i.e., smokers cannot help but smoke). The aggregate endowment of the consumption good in the economy is given by $e = 1$. The only decisions involve the allocation of consumers across restaurants and the allocation of consumption across consumers.

Question 2:

(a) Define this economy in the style of Debreu.

(b) Define an equilibrium.

(c) Provide a condition on v that implies that in equilibrium there exist restaurants that serve one smoker and one nonsmoker, i.e., the equilibrium *does not* sort the two types into different restaurants. Solve for the equilibrium in this case (use the price of the consumption good as the numeraire).

(Hint: The Welfare Theorems apply. Start by considering under which circumstances such an allocation would be Pareto optimal. Then, solve for the prices associated with visiting different types of restaurants from the utility maximization problem of the group that is indifferent between two types of restaurants in equilibrium).

(d) Briefly explain why the negative effect imposed by smokers on nonsmokers *does not* lead to economic inefficiencies (no formal analysis required).