Economics 201A, Final Review Section — Anton Cheremukhin

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1. Final 2005 Question 3

3. Portfolio Choice

Consider an economy with 2 assets and 3 states. The return vector for asset 1 is $z_1 = (16, 24, 32)$. The return vector for asset 2 is $z_2 = (0, 12, 32)$. Each individual has a VNM utility function $v(c^h) = (c^h)^{1/2}$. There are H individuals and each begins with an equal share of the two assets. The probability vector is $\pi = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$.

(a) Solve for the Walrasian Equilibrium state claims prices and portfolio of each individual. What are the asset prices in this economy.

(b) If individuals can trade only in asset markets, each can purchase a portfolio that is a fraction of the aggregate endowment. Does it follow that the outcome is the same as in part (a)? Would the answer be different if the different individuals had different initial shares of each asset?

(c) "If there are fewer asset markets than state claims, there is less opportunity for risk-spreading with trade only in asset markets." Can this statement be reconciled with your answer to (b).

(d) Suppose that the owners of the first asset have a production decision. If they choose θ, the asset 1 return is $z_1(\theta) = (16\theta^2, 24, 32(2-\theta))$. What is the WE level of θ? What is the efficient level of θ?

Solution:

a) $U = \frac{1}{4}\sqrt{c_1} + \frac{1}{4}\sqrt{c_2} + \frac{1}{2}\sqrt{c_3}$ $\quad z_A = (16, 24, 32) \quad z_B = (0, 12, 32)$

Assume $w = z_A + z_B = (16, 36, 64)$

RA: $\frac{p_2}{p_1} = MRS = \frac{\frac{1}{4\sqrt{c_2}}}{\frac{1}{4\sqrt{c_1}}} = \sqrt{\frac{w_1}{w_2}} = \frac{2}{3}$ $\quad \frac{p_3}{p_1} = MRS = \frac{\frac{1}{2\sqrt{c_3}}}{\frac{1}{4\sqrt{c_1}}} = 2\sqrt{\frac{w_1}{w_3}} = \frac{2*4}{8} = 1 \quad p = (3, 2, 3)$

$p_A = 16*3 + 24*2 + 32*3 = 192 \quad p_B = 0*3 + 12*2 + 32*3 = 120 \quad \frac{p_B}{p_A} = \frac{120}{192} = \frac{5}{8}$

b) $\max_{\{c^A, c^B\}}[\frac{1}{4}\sqrt{16c^A + 0*c^B} + \frac{1}{4}\sqrt{24c^A + 12c^B} + \frac{1}{2}\sqrt{32c^A + 32c^B} | p_A c^A + p_B c^B \leq p_A + p_B]$

$\frac{16}{4\sqrt{16c^A+0*c^B}} + \frac{24}{4\sqrt{24c^A+12c^B}} + \frac{32}{2\sqrt{32c^A+32c^B}} = \lambda p_A \quad \frac{0}{4\sqrt{16c^A+0*c^B}} + \frac{12}{4\sqrt{24c^A+12c^B}} + \frac{32}{2\sqrt{32c^A+32c^B}} = \lambda p_B$

Hence, $\frac{p_B}{p_A} = \left.\frac{\frac{0}{4\sqrt{16c^A+0*c^B}} + \frac{12}{4\sqrt{24c^A+12c^B}} + \frac{32}{2\sqrt{32c^A+32c^B}}}{\frac{16}{4\sqrt{16c^A+0*c^B}} + \frac{24}{4\sqrt{24c^A+12c^B}} + \frac{32}{2\sqrt{32c^A+32c^B}}}\right|_{(1,1)} = \frac{\frac{0}{4\sqrt{16}} + \frac{12}{4\sqrt{24+12}} + \frac{32}{2\sqrt{32+32}}}{\frac{16}{4\sqrt{16}} + \frac{24}{4\sqrt{24+12}} + \frac{32}{2\sqrt{32+32}}} = \frac{5}{8}$

It does not depend on the initial shares of individuals.

c) We have a specific case of identical homothetic preferences. In general the statement is right.

d) Efficient: $\frac{1}{4}\sqrt{16}\theta + \frac{1}{4}\sqrt{24+12} + \frac{1}{2}\sqrt{32(2-\theta)+32} \to \max_\theta \quad 1 = \frac{1}{4}\frac{\sqrt{32}}{\sqrt{3-\theta}} \quad \Leftrightarrow \quad \theta^* = 1$

WE is efficient both under the assumption of complete markets (contingent claims) and incomplete markets because the preferences are identical and homothetic, and the sum of assets delivers the total consumption vector in any case.

2. Final 2004 Question 4

4. Spot and Futures Prices

There are 2 commodities and 2 periods. Alex has an endowment vector $\omega^A = (80,140;50,90)$ where the first two components $(\omega_{11},\omega_{12})$ are his period 1 endowment and the second two components $(\omega_{21},\omega_{22})$ are his second period endowment. Bev has an endowment vector $\omega^B = (20,60,150,210)$. Both Alex and Bev have a utility function $U(c) = \ln c_{11} + \ln c_{12} + \ln c_{21} + \ln c_{22}$

(a) Solve for the Walrasian Equilibrium spot and futures prices. (Normalize and set $p_{11} = 1$.)

(b) Suppose that there is trade in both period 1 and period 2 and the period 2 spot price of commodity 1 is 1, that is the same as the period 1 price. What is the spot price of commodity 2 and what is the interest rate?

(c) Suppose that there are no futures markets. Alex and Bev trade on spot markets and save/borrow. Show that Alex saves and Bev borrows. How much does Alex save?

Solution:

a) $U = \ln c_{11} + \ln c_{12} + \ln c_{21} + \ln c_{22}$ $\quad w_A = (80, 140, 50, 90)$ $\quad w_B = (20, 60, 150, 210)$

RA: $MU = \left(\frac{1}{c_{11}}, \frac{1}{c_{12}}, \frac{1}{c_{21}}, \frac{1}{c_{22}}\right)\Big|_{c=w=w_A+w_B} = \left(\frac{1}{100}, \frac{1}{200}, \frac{1}{200}, \frac{1}{300}\right) = \left(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}\right)$ - spot and futures prices.

b) Spot prices: $\left(1, \frac{1}{2}\right)$ Future spot prices $\left(1, \frac{2}{3}\right)$

Interest rate: $\frac{1}{2} = p_{21} = \frac{p^{fs}}{1+r} = \frac{1}{1+r} \Rightarrow r = 1$

c) They consume a fraction of the total endowment proportional to their incomes.

Income of Alex $= pw_A = \left(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}\right)(80, 140, 50, 90)' = 80 + 70 + 25 + 30 = 205$

Income of Bev $= pw_B = \left(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}\right)(20, 60, 150, 210)' = 20 + 30 + 75 + 70 = 195$

Both have Cobb-Douglas utilities, and because the coefficients are equal, would spend equal incomes in both periods.

Alex's income in period 1 is 80+70=150. He is going to spend only $205/2 = 102.5$. Therefore, Alex is going to save 47.5 and that's the same amount Bev is going to borrow.

3. Final 2002 Question 5

5. 2×2 Economy

The production function in industry A is $X_A = L_A^{1/2} K_A^{1/2}$ and in industry B is $X_B = (L_B^{1/2} + K_B^{1/2})^2$. The aggregate endowment of labor is 100 and of capital is 400.

(a) Characterize the set of efficient allocations in an Edgeworth Box diagram. What happens to the wage rental ratio w/r as output of commodity A increases?

(b) What is the range of wage rental ratios that is consistent with the production of both commodities in a Walrasian equilibrium?

(c) What is the upper bound of price ratios p_X / p_Y such that both goods will be produced?

(d) If the output price vector is $(p_X, p_Y) = (6,1)$ what is the input price vector?

Solution:

a-b) $X_A = \sqrt{L_A K_A}$ $\quad X_B = \left(\sqrt{L_B} + \sqrt{K_B}\right)^2$ $\quad L_A + L_B = 100$ $\quad K_A + K_B = 400$

$$MRTS_A = \frac{\frac{1}{2}\sqrt{\frac{K_A}{L_A}}}{\frac{1}{2}\sqrt{\frac{L_A}{K_A}}} = \frac{K_A}{L_A} \qquad MRTS_B = \frac{\frac{2}{2\sqrt{L_B}}\left(\sqrt{L_B}+\sqrt{K_B}\right)}{\frac{2}{2\sqrt{K_B}}\left(\sqrt{L_B}+\sqrt{K_B}\right)} = \sqrt{\frac{K_B}{L_B}}$$

Production Efficiency implies: $\sqrt{\frac{400-K_A}{100-L_A}} = \frac{K_A}{L_A}$ This curve is below the diagonal in coordinates (L,K), because on the diagonal $MRTS_A = 4 > 2 = MRTS_B$.

$MRTS_A = MRTS_B = w/r$ in equilibrium, which is in the interval $[2, 4]$.

As output of commodity A increases, the wage/rental rate increases from 2 to 4.

c) The cost function for good A is $C(q_A) = q_A * 2\sqrt{rw}$ $\quad \Rightarrow \quad p_A = MC_A = 2\sqrt{rw}$

The cost function for good B is $C(q_B) = q_B * \frac{1}{\frac{1}{r}+\frac{1}{w}}$ $\quad \Rightarrow \quad p_B = MC_B = \frac{1}{\frac{1}{r}+\frac{1}{w}}$

Hence, $\frac{p_A}{p_B} = 2\sqrt{rw}\left(\frac{1}{r} + \frac{1}{w}\right) = 2\left(\sqrt{\frac{w}{r}} + \sqrt{\frac{r}{w}}\right) \in \left[3\sqrt{2}, 5\right]$.

In the open interval both goods are produced.

d) If $\frac{p_A}{p_B} = 6$, then only good A is produced, hence, $w/r = 4$.