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Exercise 1

4. Asset and state claims prices with more states than assets

Each individual has the same VNM utility function $v(c) = c^{1/2}$. There are 4 equally likely states. The aggregate endowment of state claims is $\omega = (1,4,9,16)$.

(a) Solve for the equilibrium state claims price vector and show that every agent will consume a fraction of the aggregate endowment.

(b) The aggregate endowment is in the form of two assets Asset A has return $z^A = (1,1,1,1)$ while asset B has return $z^B = (0,3,8,15)$. What is the WE asset price ratio?

(c) Suppose that individuals can only trade in asset prices. Is the asset price ration of part (b) still the equilibrium price ratio? If so, explain. If not, why not?

(a) There are four states. There are state claims which give a unit of good in each state. The aggregate endowment is $(1,4,9,16)$. Because the states are equally likely the maximization problem is: $\max_{\{c^A,c^B\}}[v(c^1)+v(c^2)+v(c^3)+v(c^4)|p_1c^1+p_2c^2+p_3c^3+p_4c^4 \leq p_1+4p_2+9p_3+16p_4]$

FOC: $\frac{v'(c^i)}{p_i} = \lambda$ Hence, $\frac{p_i}{p_j} = \left(\frac{c^i}{c^j}\right)^{-1/2} = \sqrt{\frac{c^j}{c^i}}$.

From the the representative-agent intuition we get the following price-vector:
$(p_1,p_2,p_3,p_4) = (1,\frac{1}{2},\frac{1}{3},\frac{1}{4})$

From the first order conditions the consumption of claims is the same function of prices for each agent: $\frac{c^j}{c^i} = \left(\frac{p_i}{p_j}\right)^2$. Hence, the consumptions of all agents are linearly dependent. As long as they sum up to the whole endowment, this means, that every agent is consuming a fraction of the endowment.

(b) Using prices of contingent claims the prices for the assets are:
$p_B = z_Bp = 0+\frac{3}{2}+\frac{8}{3}+\frac{15}{4} = \frac{95}{12}$ $\quad p_A = z_Ap = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4} = \frac{25}{12}$ $\quad \frac{p_B}{p_A} = \frac{19}{5}$

(c) Here the markets are incomplete. Hence, we have to solve the whole problem.

Now there are only two assets: A and B. Asset A gives (1,1,1,1). Asset B gives (0,3,8,15). Because the initial endowment of goods is represented by a unique linear combination of these two assets (1,1), the endowments of assets A and B are (1,1). The individual is facing the budget constraint: $p_Ac_A+p_Bc_B \leq p_A+p_B$. Because the states are equally likely the maximization problem is:

$\max_{\{c^A,c^B\}}[\Sigma_{s=1}^4 v(z_s^Ac^A+z_s^Bc^B)|p_Ac^A+p_Bc^B \leq p_A+p_B]$

FOC: $\Sigma_{s=1}^4 \frac{z_s^A}{\sqrt{z_s^Ac^A+z_s^Bc^B}} = \lambda p_A \qquad \Sigma_{s=1}^4 \frac{z_s^B}{\sqrt{z_s^Ac^A+z_s^Bc^B}} = \lambda p_B$

$$\text{Hence, } \frac{p_B}{p_A} = \left.\frac{\Sigma_{s=1}^4 \frac{z_s^B}{\sqrt{z_s^Ac^A+z_s^Bc^B}}}{\Sigma_{s=1}^4 \frac{z_s^A}{\sqrt{z_s^Ac^A+z_s^Bc^B}}}\right|_{(1,1)} = \frac{\Sigma_{s=1}^4 \frac{z_s^B}{\sqrt{z_s^A+z_s^B}}}{\Sigma_{s=1}^4 \frac{z_s^A}{\sqrt{z_s^A+z_s^B}}} = \frac{\frac{0}{\sqrt{1+0}}+\frac{3}{\sqrt{1+3}}+\frac{8}{\sqrt{1+8}}+\frac{15}{\sqrt{1+15}}}{\frac{1}{\sqrt{1+0}}+\frac{1}{\sqrt{1+3}}+\frac{1}{\sqrt{1+8}}+\frac{1}{\sqrt{1+15}}} = \frac{19}{5}$$

This is the same as with full markets. Hence the equilibrium price ratio of part (b) remains.

The result totally depends on whether the aggregate endowment can be expressed in terms of existing assets. If this was not true, one would generally get a corner solution, where not all the endowment of goods can be consumed, and hence some of it has to be "freely disposed". In this case the answer would differ from the complete-markets case.

Example: Asset A gives (1,2,3,4). Asset B gives (5,4,3,2).

In this case we first have to solve for the best endowment of these assets, so that we get maximum utility, constrained by the fact, that endowment in each state is no bigger that the actual endoment of good in that state:

$\max_{\{w^A,w^B\}}[\Sigma_{s=1}^{4} v(z_s^A w^A + z_s^B w^B)|z_i^A w^A + z_i^B w^B \leq w_i]$

If the number of states (S) is bigger than the number of assets (N), that normally N constraints will be binding and S-N constraints won't. The solution is: $w^A = \frac{13}{3}, w^B = -\frac{2}{3}$. The incomplete-market price vector will be:

$$\frac{p_B}{p_A} = \frac{\frac{1}{\sqrt{1*\frac{13}{3}+5*\left(-\frac{2}{3}\right)}}+\frac{2}{\sqrt{2*\frac{13}{3}+4*\left(-\frac{2}{3}\right)}}+\frac{3}{\sqrt{3*\frac{13}{3}+3*\left(-\frac{2}{3}\right)}}+\frac{4}{\sqrt{4*\frac{13}{3}+2*\left(-\frac{2}{3}\right)}}}{\frac{5}{\sqrt{1*\frac{13}{3}+5*\left(-\frac{2}{3}\right)}}+\frac{4}{\sqrt{2*\frac{13}{3}+4*\left(-\frac{2}{3}\right)}}+\frac{3}{\sqrt{3*\frac{13}{3}+3*\left(-\frac{2}{3}\right)}}+\frac{2}{\sqrt{4*\frac{13}{3}+2*\left(-\frac{2}{3}\right)}}} = \frac{\frac{1}{3}\sqrt{6}+\frac{3}{11}\sqrt{11}+2}{\frac{2}{3}\sqrt{6}+\frac{3}{11}\sqrt{11}+\frac{11}{2}} = 0.462\,96$$

However, in the presence of contingent claims the relative price is:

$(p_1, p_2, p_3, p_4) = (1, 1, 1, 1) \qquad \frac{p_B}{p_A} = \frac{1+2+3+4}{5+4+3+2} = \frac{5}{7} = 0.714\,29$

Exercise 2

3. Asset and state claims prices

There are 2 states and 2 assets. The asset A vector of returns in the two states is $z^A = (\alpha, \beta)$. The asset B return vector is $z^B = (1-\alpha, 2-\beta)$. The two states are equally likely. Each individual has the same VNM utility function $v(c) = \dfrac{c^{1-R}}{1-R}$, $R > 1$.

(a) Solve for the WE state claims prices.

(b) Solve for the WE asset prices.

(c) Under what conditions will the price of asset A rise relative to the price of asset B as the degree of relative risk aversion increases?

(d) Give the intuition behind this conclusion.

(e) If there are no state clams markets but individuals can trade assets, what will be the equilibrium asset prices? Explain.

(a) There are state claims which give a unit of good in each state. The individual is facing the following budget constraint: $p_1 c^1 + p_2 c^2 \leq p_1 + 2p_2$, as the aggregate endowment is (1,2). Because the states are equally likely the maximization problem is: $\max_{\{c^A,c^B\}}[v(c^1)+v(c^2)|p_1 c^1 + p_2 c^2 \leq p_1 + 2p_2]$

FOC: $\frac{v'(c^1)}{p_1} = \frac{v'(c^2)}{p_2}$ Hence, $\frac{p_2}{p_1} = \left(\frac{c^2}{c^1}\right)^{-R} = \left(\frac{c^1}{c^2}\right)^{R}$

In Walrasian Equilibrium: $\frac{p_2}{p_1} = \left(\frac{c^1}{c^2}\right)^{R}\Big|_{(1,2)} = \frac{1}{2^R}$.

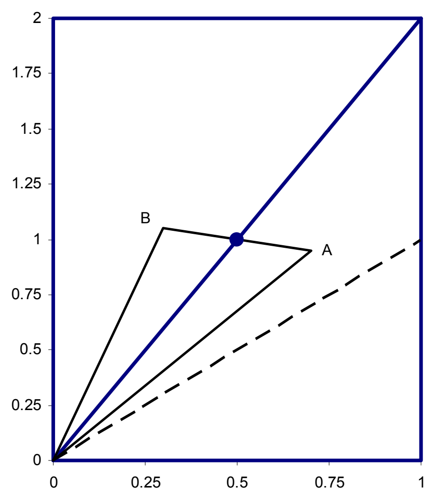
(b) Markets are complete given 2 contingent claims for 2 states of nature. Hence assets can be priced using the prices of contingent claims. Asset A gives you αp_1 and βp_2 with equal probability. Asset B gives you $(1-\alpha)p_1$ and $(2-\beta)p_2$ with equal probability. That means that

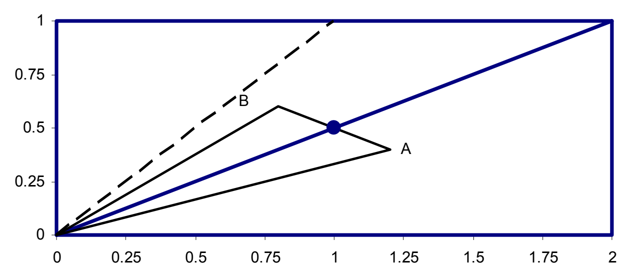
$$\frac{p_A}{p_B} = \frac{\alpha p_1 + \beta p_2}{(1-\alpha)p_1 + (2-\beta)p_2} = \frac{\alpha + \beta\frac{p_2}{p_1}}{(1-\alpha)+(2-\beta)\frac{p_2}{p_1}} = \frac{2^R\alpha+\beta}{2^R(1-\alpha)+(2-\beta)}$$

(c) The price of asset A relative to the price of asset B will rise as R rises if:

$$\frac{\partial}{\partial R}\frac{2^R\alpha+\beta}{2^R(1-\alpha)+(2-\beta)} = \frac{2^R\ln 2}{((1-\alpha)2^R+2-\beta)^2}[2\alpha-\beta] > 0 \qquad \text{i.e. } 2\alpha > \beta.$$

(d) Intuition: The PO-set is the diagonal of the Engeworth box because agents have identical homothetic preferences. The condition $2\alpha > \beta$ means that the point, representing asset A, must be below the diagonal in the Edgeworth Box. The assets sum up to the whole endowment. Hence, the point, representing asset B, must be above the diagonal in the Edgeworth Box. In this case the relative price of the asset closer to the riskless line has to grow as risk aversion grows to keep equilibrium on the diagonal. This result also depends on the fact that aggregate endoment of good 2 is bigger. In the opposite case the other asset would be closer to the riskless line. The intuition is demonstrated in the following graphs.





(e) There are two assets: A and B. Asset A gives α in state 1 and β in state 2. Asset B gives $1-\alpha$ in state 1 and $2-\beta$ in state 2. If assets are traded, the individual is facing the following budget constraint: $p_A c_A + p_B c_B \le p_A w_A + p_B w_B$. His endowment should be expressed in terms of assets. The agent owns everything if he has equal amounts of both assets equal to 1. The payoff in state 1 is $\alpha c_A + (1-\alpha)c_B$. The payoff in state 2 is $\beta c_A + (2-\beta)c_B$. Because the states are equally likely the maximization problem is:

$$\max_{\{c^A,c^B\}}[v(\alpha c^A + (1-\alpha)c^B) + v(\beta c^A + (2-\beta)c^B)|p_A c^A + p_B c^B \le p_A + p_B]$$

FOC: $\alpha v'(\alpha c^A + (1-\alpha)c^B) + \beta v'(\beta c^A + (1-\beta)c^B) = \lambda p_A$

$(1-\alpha)v'(\alpha c^A + (1-\alpha)c^B) + (2-\beta)v'(\beta c^A + (2-\beta)c^B) = \lambda p_B$

Hence, $$\frac{p_B}{p_A} = \left.\frac{(1-\alpha)\left(\frac{\alpha c^A+(1-\alpha)c^B}{\beta c^A+(2-\beta)c^B}\right)^{-R}+(2-\beta)}{\alpha\left(\frac{\alpha c^A+(1-\alpha)c^B}{\beta c^A+(2-\beta)c^B}\right)^{-R}+\beta}\right|_{(c^A=c^B)} = \frac{2^R(1-\alpha)+(2-\beta)}{2^R\alpha+\beta}.$$

Given that $\frac{\alpha}{1-\alpha} \neq \frac{\beta}{2-\beta}$ the price vector is the same as in part (b) because payoffs of the shares are independent and hence markets are complete.